# The Topography of Nations[*]

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**Abstract**

How does the interplay of geography and political economic forces together affect the shape of nations? This paper presents a quantitative framework for characterizing the equilibrium evolution of national boundaries in a world with a rich geography. The framework is based on simple equilibrium conditions based on the efficient transportation of resources that arise from disparate political economic micro-foundations. I characterize the existence, uniqueness, and efficiency of the dynamic equilibrium and provide a simple algorithm for its calculation. When combined with detailed spatial geography data from Europe, the equilibrium condiitons well approximate observed borders, and the dynamic framework is able to successfully predict the evolution of national boundaries and resulting conflict across Europe over the past millenia. Finally, I apply the framework to ask how the changing spatial distribution of resources arising from climate change may alter European borders in the future, finding that the Crimean peninsula and surrounding area is especially susceptible.

# 1   Introduction

The division of land between nations—and the conflicts over territory that often result-–have large welfare consequences. Geography plays an important role in determining national borders and oftentimes provides the justification for territorial conflict, see e.g. the "*limites naturelles*" of Revolutionary France, "Manifest Destiny" in the United States, the "*Lebensraum*" of Nazi Germany and, more recently, the annexation of Crimea by Russia. How does the interplay of geographic, political and economic forces together shape the boundaries of nations? And how do changes to that underlying geography – arising e.g. through climate change that alters the value of land or technological innovations that change the cost of transporting goods – alter the topography of nations?

This paper presents a quantitative framework for answering these questions by deriving and characterizing the equilibrium evolution of the shape of nations in a world with a rich geography. The framework is based on simple and intuitive conditions where borders are drawn in such a way to minimize internal transportation costs, which are shown to be the equilibrium conditoins that arise from multiple disparate political economic micro-foundations. I characterize a number of properties of the dynamic equilibrium including its existence, uniqueness, and efficiency, and I provide a simple algorithm for its calculation. When combined with detailed geographic data from Europe, I show that the framework both well approximates observed borders and is able to predict the evolution of national boundaries over the past millenia. Finally, I use the framework to predict how climate change may alter European borders in the future, finding that the Crimean peninsula and surrounding area is especially susceptible.

The framework asks how to take a continuous surface with a nearly arbitrary distribution of resources and transportation costs and divide it into a finite (given) number of nations. To determine this equilibrium partition, I offer two political economic micro-foundations. In the first, autocratic rulers attempt to extract as much wealth as they can by threatening war with their neighbors and collecting resources from their populace to transport to their capital. In the second, citizens freely choose their nationality and vote with their fellow citizens the location of a public good to which they must travel to enjoy its benefits.

Despite their differences, the equilibrium conditions governing the evolution of the shape of nations in both frameworks are the same. In both, the allocation of land to nations is done so to minimize transportation costs: territory is allocated to the nation where transport costs to the capital are minimized and the location of the capital is chosen to minimize the amount of resources lost in transit. These two conditions imply that the dynamic equilibrium of the framework is a (generalized) form of Lloyd's algorithm (Lloyd, 1982) and the steady state is

a (generalized) Centroidal Voronoi Tessellation (CVT) (see e.g. Du, Faber, and Gunzburger (1999)), both of which have found extensive applications in the fields of computer science and electrical engineering. As a result, I can apply results from those fields to both efficiently calculate the dynamic equilibrium and characterize its properties. For example, despite the multiplicity of steady states, I provide conditions under which the dynamic equilibrium is unique given its initial conditions. I also show how the framework can be extended in a number of directions, including introducing heterogeneity in the productivity of different nations, entry and exit of nations, and international trade.

I apply the framework to the study the evolution of national boundaries in Europe over the past millennia. To estimate the resource value of a location, I consider its potential caloric output, which evolves over time as new crop types arrive in Europe. To estimate the transportation costs between any two locations, I calculate the transportation costs incurred along the optimal route between those locations incorporating the detailed information about the local topography en route, which evolves over time as sailing technology improves and bridges are constructed across rivers.

Using this detailed geographic information, I provide two pieces of evidence suggesting that the the framework performs well empirically. First, I document that the equilibrium conditions of the framework well approximate the actual evolution of national boundaries. In particular, (1) territory tends to be allocated to the nation whose capital is nearest; and (2) the location of a capital tends to be close to the location that minimizes intranational transportation costs, as predicted by the model. Both of these facts continue to hold even when considering only variation over time within a location, e.g. from capital cities who change their locations or locations who change their nationality. Second, I assess the extent to which the framework is able to predict the evolution of national boundaries one hundred years in the future given estimates of the future geography and the borders today. I show that not only does the framework does have strong predictive power about which locations will change nationalities—a location identified by the framework as joining a particular nation has a 10% chance of actually doing so—the model also offers predictive power of where future conflicts may arise.

Finally, I apply the framework to assess how borders may evolve in the future in response to the changing spatial distribution of resources owing to climate change. Taken as given national borders in the year 2000 and the predicted change in caloric output in 2100 arising from changes in potential yields from a leading climate change model, the framework finds that the largest European region likely to change its nationality due to climate change is the Crimean peninsula and southern Ukraine. The example counterfactual illustrate the ability of the framework to quantitatively assess how changes in geography affect the shape

of nations.

This paper contributes to a number of strands of literature. It is closely connected to the political economy literature on the size and shape of nations. The focus here on an equilibrium that minimizes transportation costs is similar to the central thesis of Friedman (1977) that territory is allocated to leaders who value its potential tax revenue net of collection costs the most. Later work by e.g. Alesina and Spolaore (1997), Alesina and Spolaore (2005), Alesina, Spolaore, and Wacziarg (2005), Spolaore and Wacziarg (2005), Li and Zhang (2016), and Gancia, Ponzetto, and Ventura (2022) explicitly derive predictions on the size and shape of nations, albeit in settings with stylized geographies. In contrast, the framework here determines the equilibrium size and shape of nations on a surface with an arbitrary geographical distribution of resources and trade costs. However, unlike this earlier literature, one important limitation of the baseline version of the framework presented here is that the total number of nations is taken as given; incorporating exit and entry of nations into the framework is discussed as a model extension below.

Two closely related papers in this field are those by Weese (2016) and Fernández-Villaverde, Koyama, Lin, and Sng (2020), who conduct a large number of stochastic simulations of state formation in worlds with realistic geographies based on interactions between neighboring locations and show that these simulations match well the observed evolution of state formation in both Europe and China. Here, in contrast, I provide theoretical conditions under which the political-economic framework provides a unique evolution of nation-states for any geography given initial conditions. In this way, the paper can be seen as making a similar contribution to the political economy literature as the recent "quantitative spatial economics" models to the field of economic geography.[1]

The paper is also related to large literature examining the equilibrium location of firms in space, e.g. Eaton and Lipsey (1975), Eaton and Lipsey (1976), and Oberfield, Rossi-Hansberg, Sarte, and Trachter (2020). The dynamic technique employed here is similar to Eaton and Lipsey (1975), who characterize the properties of a variety of extensions of Hotelling (1929) based on the dynamic evolution of Voronoi tessellations, see section 7.3.1 of Boots, Okabe, and Sugihara (1999) for a discussion.

Similarly, the paper bears a resemblance to a smaller literature examining the catchment area of markets in space, e.g. Nagy (2020) and Lanzara and Santacesaria (2021). The latter paper solves for the catchment area of markets with a rich geography of trade costs using a Voronoi tessellation, which is similar to the approach taken here for determining the territory

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[1]See e.g. Allen and Arkolakis (2014), Ahlfeldt, Redding, Sturm, and Wolf (2015), Redding (2016), and the excellent review article by Redding and Rossi-Hansberg (2017) for static quantitative spatial models. Allen and Donaldson (2020) provide an example of a dynamic quantitative spatial model, where, like here, the dynamic equilibrium is unique given the initial conditions despite the possibility of multiple steady states.

of a nation given its capital city. Importantly, however, here I also allow the location of the capital city to be determined endogenously, which in turn allows the shape of a nation to evolve endogenously over time.

Finally, the paper contributes to a large body of work in political science and geography on national borders and state formation, see e.g. the excellent review article by Minghi (1963) and the book Prescott (2014). Relative to this literature, this paper provides a quantitative framework and derives the dynamic equilibrium evolution of boundaries given the underlying geography of a space.

The remainder of the paper is organized as follows. In the next section, I present the two micro-foundations, derive and characterize the dynamic equilibrium, and discuss a number of extensions to the framework. In Section 3, I apply the framework to the study of Europe over the past millenia. In Section 4, I apply the framework to predict how climate change will affect the future evolution of European borders. Section 5 concludes.

# 2 A geographic framework for explaining the shape of nations

In the following, I develop a framework for understanding how geography determines the location and shape of the nations. The goal of the framework is two-fold: first, it is meant to be flexible, i.e. it can incorporate the rich and varied topography that is characteristic of the real world; second, it is meant to be robust, delivering predictions that are consistent with multiple political economy mechanisms that may drive nation formation.

## 2.1 Setup

The world is a compact manifold $S$ in $\mathbb{R}^K$. Each point $s \in S$ is endowed with resources $\rho(s) \geq 0$. Transporting resources between locations is costly, which is modeled in an iceberg form, with a fraction $d(s, s')$ of resources sent from $s \in S$ to $s' \in S$ lost along the way. Together the functions $\rho : S \to \mathbb{R}_+$ and $d : S \times S \to [0, 1]$ comprise the *geography* of the world. I assume that (a) $\rho$ is continuous and bounded and (b) $d$ is a distance metric on $S$; apart from these restrictions, the geography is otherwise flexible.

Time $t \in \{0, 1, ...\} \equiv \mathcal{T}$ is discrete and countably infinite. At $t = 0$, $i \in \{1, ..., N\} \equiv \mathcal{N}$ different nations are endowed with a starting location for their capital, $x_{i0} \in S$; these starting locations comprise the initial conditions of the model. Note that the number of nations is taken as given; I will discuss how to incorporate the entry and exit of nations as an extension to the framework below.

Define a *nation* to be a capital location $x_{it} \in S$ and a territory $S_{it} \subseteq S$. (The purpose

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a nation serves will depend on the particular political economic micro-foundation and will be discussed below). Define a *partition* of $S$ in period $t$ to be the set of nations $\{x_{it}, S_{it}\}_{i=1}^{N}$ such (a) every capital location is in its own territory, i.e. $x_{it} \in S_{it}$ and (b) no location in the world belongs to more than one nation, i.e. $\forall i, j \in \mathcal{N}$, $\text{int} S_{it} \cap \text{int} S_{jt} = \emptyset$.

The goal of our framework is to determine the equilibrium partition for all $t \in \mathcal{T}$. To do so, I consider two distinct and disparate political economy micro-foundations, each of which turns out to yield the same equilibrium partition.

## 2.2 An autocratic "Leviathan" regime

Consider first a world comprising a set of nations whose autocratic rulers' sole goal is to establish a territory over which they can extract the maximal rents possible from the citizenry, as in Friedman (1977). Following Hobbes (1661), Alesina and Spolaore (2005) refer to such governments as "Leviathans"; Tilly (1992) contend that this view of nations approximates well much of the past millenia of European history.

Suppose that each nation is governed by a dynasty of such autocratic rulers, who I will refer to as "lords". The dynamic process of nation formation is simple. Each lord is alive for a single period and is myopic, i.e. they do not value the payoffs of future generations. Lords living in odd periods are "war-time" lords who take as given the location of their capital and compete with other lords over territory. Lords living in even periods are "peace-time" lords who take as given the territory of their nation and may choose to reallocate their capital to reduce the intra-national transportation costs they incur.

These (stark) timing assumptions deserve some discussion. Admittedly, the assumptions are made primarily for tractability: for example, they rule out the possibility of a lord choosing a capital location that delivers lower payoffs today with the expectation that it will result in greater territorial acquisitions (and resulting in higher payoffs) in the future. As will become evident, such behavior would open up the possibility of multiplicity, whereas these assumptions (under appropriate conditions) ensure the existence of a unique equilibrium. In this way, the timing assumptions can be viewed as a means of equilibrium selection. That being said, given that the evolution of national borders (and centers of power) often occurs over time spans of hundreds of years, perhaps it is reasonable to treat lords as shortsighted.

The goal of each lord is to maximize the resources he can extract from his territory, net of the transport costs of collecting those resources and any military costs incurred.[2] In particular, the returns to a lord of nation $i$ with capital $x_{it} \in S$ and territory $S_{it} \subseteq S$ is the total amount of produce of that nation net of the costs of transporting that produce to the

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[2]This assumption is similar to Friedman (1977), who posits that leaders value territory equal to the potential tax revenue net of collection costs.

capital and any military expenditures $C_{it}$:

$$R(x_{it}, S_{it}) = \int_{S_{it}} \rho(s)(1 - d(s, x_{it}))\,ds - C_{it}. \tag{1}$$

Consider a war-time lord of nation $i$ living in an odd period. That lord takes as given the location of the capital city he inherited and uses the military to lay claim to the territory of the nation that maximizes the resource extraction. I model this military process as follows. The lord lays a claim $c_{it}(s)$ on all locations $s \in S$, where $c_{it}(s)$ is the maximum amount of resources that the lord would be willing to allocate to fight for $s$. If $c_{it}(s) > 0$ and no other nation lays a strictly positive claim on a location (i.e. $c_{jt}(s) = 0 \; \forall j \neq i$), that location becomes the territory of $i$ and no costs are realized – intuitively, the military does not need to be dispatched if no enemy appears. But if two or more nations put strictly positive claims on the same location, I assume a battle occurs, where all feuding nations incur costs equal to their claims and the nation with the greatest claim winning the battle (and the location).[3] The military costs incurred are can be calculated by integrating the product of a lord's own claims and a indicator variable equal to one if any other nation makes a positive claim across all locations:

$$C_{it} = \int_S c_{it}(s) \times \mathbf{1}\left\{\max_{j \neq i} c_{jt}(s) > 0\right\} ds.$$

In order to ensure that the military claims satisfy the lord's budget constraint, I require that $C_{it} \leq \int_{S_{it}} \rho(s)(1 - d(s, x_{it}))\,ds$, i.e. $R(x_{it}, S_{it}) \geq 0$. As will become immediately clear, this constraint is not binding in the equilibrium.

I will now define a Nash equilibrium for the competition between lords over the territory.[4] If a lord believes believes that they can win the battle for location $s$, they will lay a positive claim equal to the benefit they receive from that location: A claim any smaller risks a loss in the battlefield and a claim any greater would result in negative payoffs even if the battle is won. Realizing that all other lords are behaving similarly, each lord will only attempt to lay claims on territory for which their own benefit from that location is greater than any other lord: laying claims anywhere else will result in a lost battle and wasted resources. As a result, each lord will only lay a positive claim on locations where the fraction of resources lost transporting resources to the lord's own capital is lower than all other capitals, i.e.

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[3]The equilibrium behavior of lords described in what follows depends crucially on the result of the battle depending deterministically of the claims laid by feuding nations. Introducing uncertainty into the outcome of battles would be a very interesting direction for future research.

[4]Although similar in spirit, the game defined here differs from the classic Colonel Blotto game (see e.g. Borel (1921) and Macdonell and Mastronardi (2015)) in two respects: 1) different nations value each location differently depending on the transportation costs to that location; and 2) nations may opt to not lay any claim on a location, i.e. the aggregate amount of military expenditure of a nation is not fixed.

$d\left(s, x_{it}\right) \leq \min_{j \in \{1,..,N\}} d\left(s, x_{jt}\right).$

This yields an equilibrium set of claims:

$$c_{it}\left(s\right) = \begin{cases} \rho\left(s\right)\left(1 - d\left(s, x_{it}\right)\right) & \text{if } d\left(s, x_{it}\right) \leq \min_{j \in \mathcal{N}} d\left(s, x_{jt}\right) \\ 0 & \text{otherwise} \end{cases},$$

with resulting territories:

$$S_{it} = \left\{ s \in S | d\left(s, x_{it}\right) \leq \min_{j \in \mathcal{N}} d\left(s, x_{jt}\right) \right\}. \tag{2}$$

Battles will only occur on boundary locations where there exists an $i, j \in \mathcal{N}$ such that $d\left(s, x_{it}\right) = d\left(s, x_{jt}\right)$. Since these locations are measure zero, they do not affect the lords payoffs, i.e. $C_{it} = 0$ in equilibrium.

Now consider a peace-time lord of nation $i$ living in an even period. The lord takes as given the territory of the nation $S_{it}$ and chooses the location of the capital city in order to maximize the natural resources net of the costs incurred in gathering them, i.e.:

$$x_{it} = \arg\max_{x \in S_{it}} \int_{S_{it}} \rho\left(s\right)\left(1 - d\left(x, s\right)\right) ds.$$

or, equivalently, minimize the resources lost to trade costs:

$$x_{it} = \arg\min_{x \in S_{it}} \int_{S_{it}} \rho\left(s\right) d\left(x, s\right) ds \tag{3}$$

Equations (3) and (2) determine the equilibrium partition of the world into nations for all time periods: given the location of capital cities from the previous period, boundaries are re-drawn so that the territory is assigned to the nation with the nearest capital city; then the capital city is chosen to minimize the transportation costs within the existing territory, with the process continuing until a stable partition is reached. Prior to discussing the characteristics of this evolution, however, I turn to a very different micro-foundation of the formation of nations that turns out to generate the same equilibrium evolution of nations.

## 2.3 A democratic voting regime

Consider now a world comprising a set of democratic nations who offers a public good to its citizens, as in Alesina and Spolaore (1997). The citizenry, in turn, choose which nation to be a part of and vote on the location of the public good. There are two key distinctions with the autocratic regime described above: first, rather than a nation being defined by military

competition over territory, here each (immobile) individual freely chooses which nation they are a part of; second, rather than the location of the public good (i.e. its capital) being chosen to maximize the rents extracted by the lord, here the citizenry of a nation collectively votes to determine its location.

The timing of the model is chosen to be as comparable as possible to the timing above. People live for one period and are myopic. People living in odd periods take as given the location all the public goods and choose which nation they would like to become a citizen. People living in even periods take as given their citizenship and vote for where the public good should be located. In both periods, the measure of individuals in a location $s$ is $\rho(s)$, and an individual in location $s$ who is a citizen of nation $i$ in time $t$ receives the benefit of the public good (normalized to one) net of the transportation costs necessary to enjoy it:

$$R_{it}(s) = (1 - d(s, x_{it})).$$

Consider first an individual residing in $s$ in odd $t$ who is choosing which nation to join, i.e. $\max_{i \in \mathcal{N}} R_{it}(s)$. Because the gross benefit of the public good is the same across all nations, she will simply choose her nationality to maximize her benefit net of transportation costs. In equilibrium, this will imply that the territory of the nation will be defined by the set of locations for whom the trade costs are minimized, as in equation (2).

In even $t$, each nation $i$ will hold a referendum amongst all citizens over the location of the public good. I remain agnostic about the particular process through which the choice occurs, but instead assume that the choice is an efficient one, i.e. it maximizes the sum of all citizens payoffs, i.e.:

$$x_{it} = \arg\max_{x \in S_{it}} \int_{S_{it}} \rho(s)(1 - d(s, x_{it}))\, ds,$$

or equivalently, minimizes the resources lost to trade costs, as in equation (3).[5]

Hence, despite the very different underlying assumptions concerning the role of nations, the same equations (3) and (2) that determine the equilibrium partition of the world into nations for all time periods in the autocratic "Leviathan" regime also determine the equilibrium partition for the democratic voting regime. As will become evident in the next section, this is for the simple reason that despite their obvious different, both regimes deliver equilibrium partitions that efficiently minimize transportation costs. I now turn to the characterization of these processes.

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[5]The particular process through which elections yield the efficient outcome is an interesting question but beyond the scope of this paper; see e.g. Gershkov, Moldovanu, and Shi (2017).

## 2.4 Characterizing the equilibrium evolution of nations

I now characterize the equilibrium evolution of nations. I begin by formally defining the dynamic equilibrium.

**Definition.** For any geography $\{\rho, \tau\}$ and initial set of capitals $\{x_{i0}\}_{i \in \mathcal{N}}$, a *dynamic equilibrium* is the set of partitions $\{x_{it}, S_{it}\}_{i \in \mathcal{N}, t \in \mathcal{T}}$ such that:

(a) For odd $t \in \{1, 3, ...\}$, territory $\{S_{it}\}_{i \in \mathcal{N}}$ solves:

$$S_{it} = \left\{ s \in S | d(s, x_{it-1}) \leq \min_{j \in \mathcal{N}} d(s, x_{jt-1}) \right\}. \tag{4}$$

(b) For even $t \in \{2, 4, ...\}$, capitals $\{x_{it}\}_{i \in \mathcal{N}}$ solves:

$$x_{it} = \arg \min_{x \in S_{it-1}} \int_{S_{it-1}} \rho(s) d(s, x) ds. \tag{5}$$

Of particular interest in what follows will be the steady state of the dynamic equilibrium, which is defined as:

**Definition.** For any geography $\{\rho, \tau\}$, a *steady state* is the set of partitions $\{x_i^*, S_i^*\}_{i \in \mathcal{N}}$ such that:

(a) The steady state territory solves:

$$S_i^* = \left\{ s \in S | d(s, x_i^*) \leq \min_{j \in \mathcal{N}} d(s, x_j^*) \right\}. \tag{6}$$

(b) The steady state capitals solves:

$$x_i^* = \arg \min_{x \in S_i^*} \int_{S_i} \rho(s) d(s, x) ds. \tag{7}$$

In the special case where $d(s, x) \equiv \|s - x\|_2^2$, the dynamic equilibrium corresponds to Lloyd's algorithm (Lloyd, 1982) and the steady state equilibrium is known as a Centroidal Voronoi Tessellation (CVT) (see e.g. Du, Faber, and Gunzburger (1999)), which both have a large number of applications across computer science and electrical engineering, including data compression, image processing, signal processing, and cluster analysis.[6] As a result, a number of their properties have been established. While less progress has been made for more general metrics (see e.g. Ye, Yi, Yu, Liu, and He (2019)) such as the one considered

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[6]CVTs even show up in the natural world; see e.g. Barlow (1974) for an example of the territory of the male Tilapia mossambica.

here, in what follows I build off this large literature to characterize the existence, uniqueness, convergence, and efficiency of the model.

### 2.4.1 Existence and uniqueness

I first begin by providing conditions under which the equilibrium exists and is unique through the following two propositions:

**Proposition 1.** *For any geography $\{\rho, d\}$ and capital cities $\{x_{i,t-1}\}_{i \in \mathcal{N}}$, there exists unique territories $\{S_{it}\}$ satisfying equation (4).*

*Proof.* See Online Appendix A.1.1. □

Given any $\{x_{it-1}\}$ the set of $S_{it}$ defined by (4) defines what is known as a *Voronoi diagram* of $S$, see e.g. Boots, Okabe, and Sugihara (1999). Its existence and uniqueness has been well established, making Proposition 1 – i.e. the evolution of an equilibrium in odd periods given the previous even period – straightforward.

The evolution of an equilibrium in even periods given the previous odd period is more difficult. While establishing the existence of solutions to (5) is straightforward, the conditions under which such a *Fréchet mean* is unique remains an active area of inquiry, see e.g. Afsari (2011)).[7] In what follows, we establish uniqueness under the following regularity condition:

**Condition 1.** The transport cost function $d$ is such that for any $x, y, s \in S$, there exists a $\lambda \in (0, 1)$ such that $d(s, \lambda x + (1 - \lambda) y) \leq \lambda d(s, x) + (1 - \lambda) d(s, y)$, with the equality strict almost everywhere in $S$.

Note that Condition 1 holds in the special case where $d(x, y) = \|x - y\|_2^2$. In this special case, the Fréchet mean has an explicit solution as the *centroid* of $S_{it-1}$, i.e. $x_{it} = \int_{S_{it-1}} s\rho(s) \, ds / \int_{S_{it-1}} \rho(s) \, ds$. For a more general metric (such as the optimal routing metric over a terrain of varying trade costs I employ below), I am unaware of a straightforward method of verifying Condition 1; however, in practice, a simple gradient search algorithm quickly finds the Fréchet mean.

**Proposition 2.** *For any geography $\{\rho, d\}$, there exists a $\{x_{it}\}_{i \in \mathcal{N}}$ satisfying equation (5). If Condition 1 holds and $\{S_{it-1}\}_{i \in \mathcal{N}}$ are convex, the $\{x_{it}\}_{i \in \mathcal{N}}$ satisfying equation (5) are unique.*

*Proof.* See Online Appendix A.1.2. □

Together, Propositions 1 and 2 characterize the existence and uniqueness of the dynamic equilibrium.

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[7]Although centroids technically refer to the special type of Fréchet mean corresponding to the metric $d(s, x) \equiv \|s - x\|_2^2$ where $\rho$ is uniform, I will refer to the solution of equation (5) as a "centroid" in what follows.

### 2.4.2 Efficiency and convergence

Define $R_t$ as the *aggregate welfare* net of transportation costs, i.e.:

$$R_t \equiv \sum_{i \in \mathcal{N}} \int_{S_{it}} \rho(s) \left(1 - d(x_{it}, s)\right) ds.$$

The following proposition ensures that aggregate welfare always improves over time:

**Proposition 3.** *For any geography and* $\{x_{i0}\}_{i \in \mathcal{N}}$, *aggregate welfare improves over time, i.e.* $R_t \geq R_{t-1} \ \forall t$.

*Proof.* See Online Appendix A.1.3. □

Because the dynamic equilibrium is characterized by always improving aggregate welfare over time, one might expect that it is possible to show that the dynamic equilibrium converges to some steady state that (locally) maximizes aggregate welfare. This result – due to Sabin and Gray (1986) – is true under the following condition:

**Condition 2.** The transport cost function $d$ is such that for all $\lambda \in (0, 1)$ and for any $x, y, s \in S$, $d(s, \lambda x + (1 - \lambda) y) \leq \lambda d(s, x) + (1 - \lambda) d(s, y)$.

Note the close relationship between Conditions 1 and 2. Because of the possibility of multiple steady states (more on that below), a little more notation is required before stating the proposition. Let $\Gamma$ be the set of steady state capitals $\{x_i^*\}_{i \in \mathcal{N}}$. For any $\mathbf{x} \equiv \{x_i\}_{i \in \mathcal{N}} \in S$, let $\sigma(\mathbf{x}, \Gamma) \equiv \inf_{\mathbf{y} \in \Gamma} \sum_{i \in \mathcal{N}} d(x_i, y_i)$ be the distance between $\mathbf{x}$ and the set $\Gamma$. Let $R(\mathbf{x}) \equiv \sum_{i \in \mathcal{N}} \int_{S_i(\mathbf{x})} \rho(s) (1 - d(x_i, s)) ds$ be the aggregate welfare generated by the set of capitals $\mathbf{x}$ (given the unique territories induced by the capitals from equation (4)).

**Proposition 4.** *(Sabin and Gray, 1986) If Condition 2 holds, then for any geography and* $\{x_{i0}\}_{i \in \mathcal{N}}$, *the dynamic equilibrium* $\mathbf{x}_{it} \equiv \{x_{it}\}_{i \in \mathcal{N}}$ *approaches the set of steady states, i.e.* $\lim_{t \to \infty} \sigma(\mathbf{x}_{it}, \Gamma) = 0$. *Moreover, the aggregate welfare of the dynamic equilibrium converges to the aggregate welfare of one of the steady states, i.e.* $\exists \mathbf{x}^* \in \Gamma$ *such that* $\lim_{t \to \infty} E(\mathbf{x}_{it}) = E(\mathbf{x}^*)$.

Proposition 4 says that the capital cities of the dynamic equilibrium will approach the set of steady state capitals and its aggregate welfare will approach the aggregate welfare of a steady state. Clearly, if there is a unique steady state, then Proposition 4 guarantees that the dynamic equilibrium will converge to that steady state. However, the multiplicity of steady states of a common occurrence; for example, Urschel (2017) shows that with $N = 2$ and $d(x, y) = \|x - y\|_2^2$, there will be multiple steady states for any $\rho$. Propositions 1 and 2 then

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highlight the sort of path dependence emphasized in e.g. Allen and Donaldson (2020), where the dynamic equilibrium is unique but the initial conditions of history (i.e. the configuration of initial capital cities $\{x_{i0}\}_{i \in \mathcal{N}}$) converge to different long-run steady states.

The following two propositions summarize the efficiency properties of the steady state:

**Proposition 5.** *Any steady state is locally efficient, i.e. deviations from either $S_i^*$ or $x_i^*$ holding the other constant cannot improve the aggregate welfare.*

*Proof.* See Online Appendix A.1.4. □

Finally, I relate the steady state of the framework here to the planner's problem which chooses the steady state partition to maximize aggregate welfare. The following argument follows Du, Faber, and Gunzburger (1999). Define the *globally efficient partition* $\{\mathbf{x}^{**}, \mathbf{S}^{**}\} \equiv \left\{ [x_i^{**}]_{i \in \{1,\ldots,N\}}, [S_i^{**}]_{i \in \{1,\ldots,N\}} \right\}$ as the solution to the planner's problem:

$$\mathbf{x}^{**} \equiv \arg \max_{\mathbf{x} \in S^N} \sum_{i \in \mathcal{N}} \int_{S(\mathbf{x})_i} \rho(s) \left(1 - d(x_i, s)\right) ds$$

$$\mathbf{S}^{**} \equiv S(\mathbf{x}^{**}),$$

where $S(\mathbf{x})_i$ is the territory for nation $i$ induced by the capital city locations $\mathbf{x}$ from equation (7). That is, the solution to the planner's problem is the set of capital cities that induces the smallest loss of resources to transportation costs. It can be shown that $\sum_{i \in \mathcal{N}} \int_{S(\mathbf{x})_i} \rho(s) d(x_i, s) ds$ is continuous, so since $S^N$ is compact, there exists a solution to the planners problem.[8] We then have:

**Proposition 6.** *The globally efficient partition is a steady state.*

*Proof.* By definition, $\{S_i^{**}\}$ satisfy equation (7) given $\{x_i^{**}\}$, so all the remains to show is that $\mathbf{x}^{**}$ satisfy equation (6). This is established in Proposition 3.2 of Du, Faber, and Gunzburger (1999). □

Propositions 5 and 6 formally establish why the steady state of the framework developed here is "robust" to very different political-economic micro-foundations. Proposition 5 says that any steady state of this framework (locally) maximizes aggregate welfare. Proposition 6 says that the partition that maximizes aggregate welfare is a steady state of this framework. As a result, any political-economic framework whose equilibrium is efficient—i.e. it maximizes aggregate welfare—will be a steady state for the framework here as well. Or, to put it another way, while the autocratic "Leviathan" regime and the democratic voting regime are

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[8]See Section 3.2 of Du, Faber, and Gunzburger (1999).

polar opposites in their implications for the distribution of resources, the two are identical from an efficiency perspective. And as a result, the two deliver identical predictions for the evolution of the shape of nations.[9]

## 2.5 Examples

To provide some intuition about the dynamic equilibrium of the framework, I proceed by offering several examples.

### 2.5.1 Line

Consider the unit interval with two countries and suppose that $d(x, y) = |x - y|^2$. Let $b_t$ denote the border between the two nations in period $t$ (so that $S_{1t} = [0, b_t]$ and $S_{2t} = [b_t, 1]$). Then equations (4) and (5) defining the dynamic equilibrium become:

$$b_t = \frac{c_{1,t-1} + c_{2,t-1}}{2} \text{ for } t \text{ odd,}$$

$$c_{1,t} = \frac{\int_0^{b_{t-1}} x\rho(x)\, dx}{\int_0^{b_{t-1}} \rho(x)\, dx}, \quad c_{2,t} = \frac{\int_{b_{t-1}}^1 x\rho(x)\, dx}{\int_{b_{t-1}}^1 \rho(x)\, dx} \text{ for } t \text{ even,}$$

i.e. the boundary is always evenly placed between the two capitals and given the boundary, each nation places its capital at its center of mass determined by $\rho$. In the simple case where $\rho(x)$ is constant across the line interval, the steady state is simply an equal partition of the space, with $c_1 = \frac{1}{4}$, $c_2 = \frac{3}{4}$, and $b = \frac{1}{2}$. Note the difference between this equilibrium and that of Hotelling (1929) (where the two firms choose to co-locate at $\frac{1}{2}$): here, the threat of force enables the countries to locate their capitals to minimize trade costs while still keeping their territory secure; moreover, the lack of forward looking behavior prevents agents strategically choosing their capital in the hopes of capturing more territory in the future.[10]

What happens if resources are heterogeneous? Consider the simple case where $\rho(x) = x$. Then the steady state system of equations becomes $c_1 = \frac{2}{3}b$, $c_2 = \frac{2}{3}\frac{(1-b^3)}{(1-b^2)}$, and $b = \frac{c_1+c_2}{2}$, which has a solution $b = 0.6180$, $c_1 = 0.412$, and $c_2 = 0.824$, i.e. the more research rich country (country 2) is smaller. Intuitively, country 2 chooses to move its capital city towards

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[9]It is important to note, that this statement takes as given the number of nations in the world. As Alesina and Spolaore (2005) show, different regimes may result in different equilibrium number of nations. I return to this when I consider entry and exit of nations below.

[10]See Eaton and Lipsey (1975) for a comprehensive treatment of the conditions under which Hotelling's "principle of minimum deviation" apply. That the threat of force overturns it is similar to their demonstration that the principle of minimum deviation relies crucially on the assumption that firms assume other firms will not change their location when making their own location decision, i.e. the assumption of zero conjectural variation.

its greater resources, allowing country 1 to expand its territory. Panels (a), (b), and (c) of Figure 1 provides an illustration of the dynamic path that leads to this steady state, with red stars indicating the previous period's capital cities, yellow stars indicating the current period's capital cities, and the blue and yellow indicating country 1 and country 2's current territory, respectively.

How do heterogeneous trade costs affect the equilibrium? One possibility is that they can form "natural" borders. Suppose the resources remain heterogeneous with $\rho(x) = x$ but now there is a "mountain range" (i.e. high $\tau(x)$) in the middle of the line interval. If traversing the mountain range is sufficiently costly, then in the steady state the border may remain at $\frac{1}{2}$, despite the resource heterogeneity. Panel (d) of Figure 1 provides an illustration. Note that while the borders remain the same as in the homogeneous case, the capital cities in both countries are to the right of center in order to locate closer to the high resource areas. Note too that the "mountains" create the possibility of multiple steady states: in addition to the one depicted in Panel (d) of Figure 1, depending on the initial location of the two capital cities, the dynamic equilibrium may converge to a steady state with both countries located on the same side of the mountain.

### 2.5.2 The Plane

The possibility of multiple steady states becomes more pronounced of an issue when we consider two-dimensional settings. Even in the case where resources and trade costs are both homogeneous, different initial distributions of capital cities $\{x_{i0}\}$ converge to many different possible steady states. Panels (a) and (b) of Figure 2 illustrate two possible configurations for a square subset of the two-dimensional plane with 25 different nations and $d(x,y) = \|x - y\|_2^2$. If the initial distribution of capital cities $\{x_{i0}\}$ are arrayed on a grid, they converge to a steady state where each nation is a square (panel a); if the initial distribution if chosen randomly, they converge to a steady state like the one illustrated in panel (b). These different configurations illustrate the importance of the initial conditions in selecting the steady state.

What happens if resources are heterogeneous? Panel (c) of Figure 2 illustrates how the steady state partition changes from the homogeneous case to one where resources are increasing in the direction of the north-east of the surface. As with the line, nations in the regions with greater resources shrink relative to the case where resources are homogeneous, although how the partition changes, the new resulting steady state its the efficiency differ depending on the initial distribution of capital cities.

How do heterogeneous trade costs affect the equilibrium partition? Panel (d) of Figure 2 illustrates how the steady state partition changes when there is a costly to traverse river flowing from the southwest to the northeast of the figure. As with the line, the river creates

a "natural border" between nations. Note that because the river itself meanders, this results in non-convex national territories.

## 2.6    Extensions

In this subsection, I describe how the baseline framework can be extended to incorporate heterogeneous productivity across nations, nation entry and exit, and international trade. In Online Appendix A.2, I describe four additional extensions: economies of scale, unclaimed territories, multiple markets, and heterogeneous goods.

### 2.6.1    Heterogeneous productivity across nations

In the baseline model, all nations are assumed to be equally productive in transporting resources. In reality, however, it is reasonable to expect that differences in capabilities of rulers, capacity of the state, quality of institutions, ability to provide public goods like infrastructure, etc. vary across different nations. The following extension offers a straightforward way of introducing such heterogeneity in the productivity of the state into the framework above.

Suppose that nation $i$ in year $t$ has productivity $\varphi_{it} \geq 1$ such that the resource lost to transporting goods from $s \in S_{it}$ to $x \in S_{it}$ is $d_i(s,x) \equiv d(s,x)/\varphi_{it}$, i.e. more productive states incur lower transportation costs. The baseline model can be viewed as the special case of this extension where $\varphi_{it} = 1$ for all $i \in \mathcal{N}$ and $t \in \mathcal{T}$. Equation 4 of the dynamic equilibrium then becomes:

$$S_{it} \equiv \left\{ s \in S \mid \frac{d(s, x_{it-1})}{\varphi_{it}} \leq \min_{j \in \mathcal{N}} \frac{d(s, x_{jt-1})}{\varphi_{jt}} \right\}, \tag{8}$$

i.e. more productive states are able to expand their territory at the expense of less productive states.

In the special case where $d(s,x) \equiv \|s - x\|_2^2$, equation (8) generates what is known as a *multiplicatively weighted Voronoi diagram*; see e.g. Section 3.1.1 of Boots, Okabe, and Sugihara (1999). The resulting territories generated need not be convex and there is even the possibility that less productive nations are entirely contained within the boundaries of more productive nations. Because Proposition 2 requires the convexity of the territories, this admits that the dynamic equilibrium may not be unique.

What determines a nations' productivity? One possibility is that past investments made in infrastructure or state capacity can result in improved productivity today. If the size of a nation affects its ability to make such investments, one would expect that its historical size may affect its contemporaneous productivity. Below, I empirically estimate the following

simple relationship between productivity today and the size of a nation in the previous period:

$$\ln \varphi_{it} = \varepsilon \ln \left\| S_{it-1} \right\|, \tag{9}$$

where $\left\| S_{it} \right\| \equiv \int_{S_{it}} ds$ is the size of a nation. As will become evident, I find that nations that are larger a century prior are more productive today, and that incorporating such heterogeneity in productivity improves the ability of the framework to explain the observed evolution of national boundaries.

### 2.6.2 Entry and exit of nations

In the baseline model, the number of nations is taken as given. In reality, however, new nations may arise or old nations may exit. Both nation entry and exit can be incorporated into the framework as follows.

Suppose a nation must incur a fixed cost $f_e$ every period. Such fixed cost could be interpreted as the cost of maintaining a military (in the autocratic regime) or the cost of providing a public good (in the democratic regime). If a nation is unable to collect sufficient resources net of transportation costs to cover the fixed cost, it would be forced to exit, i.e. a nation will exit if:

$$\int_{S_{it}} \rho\left(s\right)\left(1 - d\left(s, x_{it}\right)\right) ds < f_e.$$

Nation entry could be modeled similarly. Suppose in each location $s \in S$ there exists a latent number of "revolutionaries" who are willing to establish a new nation with a capital city in their location. They will successfully enter and become a new nation if the resources their new nation is able to collect are sufficient to cover the fixed cost, i.e. a nation $i^{new}$ enters if there exists an $x_{i^{new}t} \in S$ such that:

$$\int_{S_{i^{new},t}} \rho\left(s\right)\left(1 - d\left(s, x_{i^{new}t}\right)\right) ds > f_e,$$

where $S_{i^{new},t} \equiv \{s \in S | d\left(s, x_{i^{new}t}\right) \leq \min_{i \in \mathcal{N}} d\left(s, x_{it}\right)\}$. Of course, one could further extend the model to allow for the fixed cost for exit and entry may differ. While exit does not affect the equilibrium properties of the model (as one would take as given the location of the capital cities of the remaining nations in the subsequent period), the possible existence of multiple new national capitals $x_{i^{new}t}$ whose $S_{i^{new}t}$ overlap would require specifying which potential entrants succeed in entry. One option would be to assume that potential entrants bid for the right to enter so the $x_{i^{new}t}$ with the greatest $\int_{S_{i^{new},t}} \rho\left(s\right)\left(1 - d\left(s, x_{i^{new}t}\right)\right) ds$ is the successful entrant.

If the fixed cost $f_e$ differs depending on whether nations are autocratic or democratic, then the equilibrium number of nations (and hence their size and shape) will depend on the form of governance as well. For example, suppose that the fixed cost of autocratic nations is greater than the fixed cost of democratic nations (e.g. it is more costly to maintain a military and domestic police force necessary to sustain an autocratic regime than it is to provide the public goods on which the democratic regime is based). Then an autocratic partition will have fewer nations in equilibrium than a democratic regime, as in Alesina and Spolaore (1997) and Alesina and Spolaore (2005).

### 2.6.3 International trade

In the baseline model, resources are only transported intranationally. One simple way of extending the framework to incorporate international trade is the following. Suppose that a nation consumes a fraction $\alpha \in [0, 1]$ of resources within its own territory and "exports" a fraction $\beta \in [0, 1]$ of resources to other nations (where perhaps $\alpha + \beta > 1$ to reflect gains from trade).

How would this alter the the the the autocratic equilibrium of Section 2.2? Lords in odd periods fighting over territory would reduce their claims over territory to account for both the fact that (1) they would only capture $\alpha$ of the resources should their claim be successful; and (2) should their claim not be successful, they would still receive $\frac{\beta}{N-1}$ of the resources, so that:

$$
c_{it}(s) = \begin{cases} \left(\alpha - \frac{\beta}{N-1}\right) \rho(s)(1 - d(s, x_{it})) & \text{if } d(s, x_{it}) \leq \min_{j \in \{1,..,N\}} d(s, x_{jt}) \\ 0 & \text{otherwise} \end{cases}.
$$

While this would have no affect on the resulting allocation of territory to nations (i.e. equation (2) would remain identical), less costs would be incurred at borders, consistent with the conventional wisdom that free trade reduces conflict.[11]

International trade would, however, affect the choice of capital in even periods, becoming:

$$
x_{it} = \arg\max_{x \in S_{it}} \alpha \int_{S_{it}} \rho(s)(1 - d(s, x_{it}))\, ds
$$
$$
+ \frac{\beta}{N-1} \sum_{j \neq i} \int_{S_{jt}} \rho(s)(1 - d(s, x_{it}))\, ds,
$$

i.e. lords would now also account for the international distribution of economic activity when making their capital location decision.

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[11]See Martin, Mayer, and Thoenig (2008) for a more complete treatment on the topic.

# 3 The evolution of European borders, 1000-2000

I now apply the framework developed above to assess its ability to explain the observed evolution of the shape of European nations over the past millennium. There is no doubt that there are many forces that shaped this evolution many of which (national identity, religion, coalition formation, royal succession, etc.) are absent the simple framework presented above. The purpose of this section is not to demonstrate that these forces did not matter; rather, it is to ask whether the force central to the model above—namely the successive improvement of the efficiency of national boundaries in reducing transportation costs—also plays an important role. I contend that the answer is yes. In particular, I demonstrate two things: (1) the equilibrium conditions of the framework well approximate the observed evolution of European borders and capital over this period; and (2) the framework has predictive power of the future evolution of national boundaries and conflicts.

## 3.1 Empirical context

The empirical context is greater Europe from 1000AD to 2000AD. It is impossible to try to summarize succinctly the evolution of state boundaries for such a large region over such a long time frame with a high degree of accuracy, but in broad strokes, Europe of 1000AD was divided into a large number of loosely connected states that provided only the most cursory of public goods to their citizens. As Tilly (1992) writes, there was an "enormous fragmentation of sovereignty then prevailing throughout the territory that would become Europe. The emperors, kings, princes, dukes, caliphs, sultans and other potentates of AD 990 prevailed as conquerors, tribute-takers, and rentiers, not as heads of state that durably and densely regulated life within their realms" (p.39). Over the next seven hundred years, rulers consolidated their power and borders became better defined, but the purpose of the nation—namely rent extraction of its citizens and territorial competition between neighboring nations—remained well approximated by the simple autocratic "Leviathan" framework described above. From 1800 onwards, the rise of the nation state and republicanism suggests an increased empirical relevance of the democratic voting framework described above, although conflict between nations over territory continues until today.

## 3.2 Data

This section provides a brief description of the data; please see Online Appendix ?? for details. I begin by defining $S$ as the (planar projection) of all of Europe (and part of Northern Africa), which I discretize into 10 kilometer by 10 kilometer cells, each of which

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I interpret as a location $s$. For each $s \in S$, I collect data on its geography and its political affiliation for every century.

### 3.2.1 Nations

For each century and each land location, I identify its nationality based on the "sovereign states" layer from the digital GIS shape-file of purchased from Euratlas (Nussli, 2010). The one exception is that I consider all principalities of the Holy Roman Empire as a single sovereign state. The borders of the nations in all centuries are depicted in yellow in Figure 3.

### 3.2.2 Capitals

I identify the capital of nation as the largest city in its border during that century. To measure city size, I combine data form two sources. Euratlas (Nussli, 2010) reports the importance of a city as a value between 1 and 5; the HYDE 3.2 dataset (Klein Goldewijk, Beusen, Doelman, and Stehfest, 2017) estimates the population density across the entirety of the nation. I identify as the capital the city that maximizes the product of the (log) population density according to HYDE and its city importance in Euratlas plus a small constant. The capitals of all nations in all centuries are depicted as green stars in Figure 3.

### 3.2.3 Resources

For most of the past millennium, agriculture was the primary sector of the economy; as Tilly (1992) notes, "until very recently most of the world's agrilcutural areas, including those of Europe, were too unproductive to permit much more than a tenth of the nearby population to live off the land" (p.18). It seems reasonable, then, to approximate the resources $\rho_t(s)$ of location $s$ in time $t$ by the potential caloric yield of that land. To do so, I follow previous literature (see e.g. Nunn and Qian (2011)) by estimating historical potential yield of a particular crop with the rainfed, low input, no fertilizer yields from the Global Agro-Ecological Zones v4.0 (GAEZ) dataset assembled by the Food and Agricultural Organization of the United Nations. I then calculate the potential caloric yield of a location by taking the maximum calories that could be produced across all crops that were available in Europe in a given century.[12] Because locations varied in their relative productivity across crops,

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[12]The crops available for cultivation in the year 1000 were alfalfa, banana, cabbage, chickpea, oat, olive, onion, rye, sugarcane, and wheat. Buckwheat and rapeseed were introduced circa 1400; maize, sunflower, sweet potato, white potato, and tomatoes were introduced circa 1500; soybean was introduced circa 1700; and sugar beet was introduced circa 1800.

the introduction of new crops resulted in changes in the spatial distribution of resources, as depicted in Figure 4.

### 3.2.4 Transportation costs

To estimate the transport cost function $d$, I follow Allen and Arkolakis (2014) by assuming that goods are shipped along the fastest path, where the time it takes to pass through a location, $\tau(s)$ depends on its local topography. In particular, the travel time incurred in transporting resources from $x \in S$ to $y \in S$ is determined by:

$$T(x,y) = \inf_{g \in \Gamma(x,y)} \int_0^1 \tau(g(t)) \left\| \frac{dg(t)}{dt} \right\|_2 dt, \tag{10}$$

where $g : [0,1] \to S$ is a path and $\Gamma(x,y) \equiv \{g \in C^1 | g(0) = x, g(1) = y\}$ is the set of all possible continuous and once-differentiable paths that lead from location $x$ to location $y$.

To construct the travel cost time function $\tau$, I proceed as follows. For overland off-road travel, I estimate the travel time using Naismith's rule (Naismith, Stobinian, and More, 1892) that a person can travel 5 km/hr but it takes an additional hour for every 600m of elevation change. For overland on-road travel (where the road network is the Roman road network), I follow Davey, Hayes, and Norman (1994), who estimate that travel times are reduced by 40% on-road relative to off-road.

Travel times via water is more difficult to estimate. When traversing a river, I assume that there is a bridge or ferry crossing if there is a city within 20 kilometers. In the absence of a bridge or ferry, traversing large rivers required finding a ford, the location of which could vary; in these cases, I assume it takes a day (12 hours) to find the nearest ford and cross. As a result, changes in the number and location of cities over time results in changes in over-land routes and travel time.

For sea travel, I calculate the average strength of prevailing winds for all ocean and sea locations and assume that sailing technology has a simple time-varying linear relationship with the average wind strength:

$$speed_t(s) = \alpha_t + \beta_t wind(s).$$

I choose the technology parameters $\alpha_t$ and $\beta_t$ through a combination of calibration and estimation. For the year 1000, I set the intercept equal to 3.5 kilometers / hour, which coincides to estimates from Whitewright (2012) that in unfavorable conditions (e.g. against the wind) the velocity made good of a boat was less than 2 knots. I then use the CLIWOC log-book data from García-Herrera, Können, Wheeler, Prieto, Jones, and Koek (2005) to estimate

the relationship between speed of travel and wind strength to estimate the coefficients $\alpha_t$ and $\beta_t$ for each 25 year period between 1750 and 1850. I assume that $\beta_t$ remained constant between 1000 and 1750, linearly interpolating $\alpha_t$ over the period. Finally, with advent of the steam engine, I set $\beta_t = 0$ for the 1900 and 2000, setting the intercept $\alpha_t$ equal to 8 knots (14.8 km/hr) in 1900 and 12 knots (22.2 km/hr) in 2000. The resulting sailing technology estimates are depicted in Online Appendix Figure B.1.

Figure 5 depicts the resulting estimated speed of travel for Europe for a sample of centuries. I then calculate the total travel time $T(x, y)$ between any two locations $x, y \in S$ by solving equation (10) using the Fast Marching Method (see Tsitsiklis (1995) and Sethian (1996)). The final step is to construct the mapping between travel time, $T(x, y)$, and the fraction of resources lost in transportation, $d(x, y)$. I assume that the iceberg trade costs are proportional to travel time, so that $d(x, y) = \kappa T(x, y) / (1 + \kappa T(x, y))$. To determine the proportionality, I assume that one day (12 hours) worth of travel results in a 10% loss in the value of the resources, i.e. $\kappa = (1/12) \times (0.1/(1-0.1))$.[13]

It should be emphasized that the constructed $\rho$ and $d$ functions, by abstracting from a number of margins (and relying on imperfect estimates) are only as proxies of the true resources and transportation costs. To the extent that they differ from the true resources and transportation costs, we should expect that they will increase the difference between the model predictions and what is observed empirically. Given their imperfect nature, it is all the more surprising how well the framework is able to fit the data, a point to which I now turn.

## 3.3 Three stylized facts

Before turning to testing the framework presented above, I first present three stylized facts that are consistent with the model predictions.

### 3.3.1 Stylized Fact #1: Borders tend to be equidistant between neighboring capitals

First, according to equilibrium condition (4), any point along a border ought to be equidistant to its two neighboring capitals. To see if this true empirically, for every point $b$ on the border between nations $i$ and $j$ in year $t$, I calculate the distance from $b$ to each of its neighboring capitals and then run the following regression:

$$\ln d(s, x_{it})_{bt} = \beta \ln d(s, x_{jt})_{bt} + \ln \varphi_{it} - \ln \varphi_{jt} + \varepsilon_{bt},$$

---

[13] This 10% loss per day appears to be conservative according to historical sources; for example, Arnaud (2007) estimates a transportation cost of 0.1 denarii per 1 kilogram of wheat per day in Rome in 301 CE, which corresponds to 5/6 of its value.

where the nation-year fixed effects $\ln \varphi_{it}$ incorporate the possibility of heterogeneous productivities across nations. Panel (a) of Figure 6 plots the bin-scatter of the resulting regression results. As is evident, I find strong evidence that border locations which are further away from one neighboring capital are also further away from the other neighboring capital, consistent with the prediction of the theory.

### 3.3.2 Stylized Fact #2: Resource-rich nations tend to be smaller

Recall from examples presented in Section 2.5 that resource-rich nations tend to be smaller; intuitively, this is because locations in smaller nations are on average closer to the capital, it is more efficient for locations with high resources to be in smaller nations. To see if this true empirically, I regress the (log) average resources of a location in nation $i$ in year $t$ on the (log) size of nation $i$ in year $t$:

$$\ln \frac{1}{\left\| S_{it} \right\|} \int_{S_{it}} \rho\left(s\right) ds = \beta \ln \left\| S_{it} \right\| + \delta_i + \delta_t + \varepsilon_{it},$$

where the nation fixed effects $\delta_i$ and the year fixed effects $\delta_t$ ensures identification arises from relative changes in the size of nations over time. I exclude nations whose borders are not contained within the region of study (as I do not observe their size) and I trim the data based on the 1%/99% of average resources observed to ensure the results are not driven by outliers (e.g. by desert nations in Northern Africa with very low average resources). Panel (b) of Figure 6 depicts the bin-scatter of this regression. As is evident, consistent with the model predictions, larger nations have on average lower resources. As this pattern arises from panel variation, its suggests a move over time toward greater efficiency, a point I turn to next.

### 3.3.3 Stylized Fact #3: The topography of nations has become more efficient over time

As Propositions 4, 5, and 6 emphasize, the equilibrium of the framework is one in the locations of borders and capitals involve in such a way so as to reduce the resources lost to transportation costs over time. In reality, do we see such efficiency improvements? To assess this, I calculate the resources net of transportation costs for each location in each time period, and I regress these resources on a set of time dummies:

$$\left(1 - d\left(s, x_{it}\right)\right) \rho\left(s\right)_{sit} = \delta_t + \delta_i + \delta_s + \beta \left\| S_{it} \right\| + \varepsilon_{sit},$$

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where the country fixed effect $\delta_i$ controls for any (time-invariant) productivity differences across nations, the location fixed effect $\delta_s$ controls for the time-invariant characteristics of a location, and the nation size controls for the mechanical relationship between distance to the capital and nation size. The coefficients of interest are the time fixed effects $\delta_t$; to ensure their estimates are not being driven by improvements in transportation technology, I hold constant the transportation technology at year 1000 levels. Similarly, to ensure that the time estimates are not simply capturing aggregate improvements in resources (but still allow for the possibility that borders respond to changes in the distribution of resources), I normalize $\rho(s)$ in each year to have a mean of one. Panel (c) of Figure 6 depicts the resulting bin-scatter. As is evident, aggregate efficiency of the topography of nations is indeed improving over time, with the most notable improvement in efficiency arising between 1600 and 1900.

## 3.4 Testing the equilibrium conditions

I now turn to more direct tests of the framework by examining the extent to which the equilibrium conditions (4) and (5) are consistent with the observed evolution of the shape of nations in Europe.

### 3.4.1 Is nationality determined by the nearest capital?

The first equilibrium condition of the framework—equation (4)—makes the simple prediction that a location will be allocated to the nation whose capital is closest. To test if this is true empirically, I will take as given the observed location of capitals and ask if the observed nationality of locations is consistent with this prediction. In particular, I consider an empirical analog of equation (4), regressing a dummy variable equal to one if location $s$ in part of nation $i$ in period $t$ on the distance between $s$ and the capital $x_{it}$ and a set of fixed effects, i.e.:

$$\mathbf{1}\left\{s \in S_{it}\right\}_{sit} = \beta d\left(s, x_{it}\right) + \delta_{st} + \delta_{it} + \delta_{si} + \varepsilon_{sit}, \tag{11}$$

where the location-year fixed effect $\delta_{st}$ captures the average distance between $s$ and all nation's capitals, the nation-year fixed effect $\delta_{it}$ controls for any heterogeneity across nations in their productivities, and the location-nation fixed effect $\delta_{si}$ ensures that the identification of $\beta$ arises only from locations whose nationality has changed over time. That is, regression (11) asks if a capital city moves closer to a given location, is that location more likely to switch its nationality to that capital. The model predicts a negative $\beta$, i.e. locations that move further away from a capital should be less likely to belong to that capital's nation.

Panel (a) of Table 1 presents the results. As is evident, regardless if one measures distance as the fraction of value lost or simply as (log) travel time and regardless of which

fixed effects are included, the probability of a location sharing the nationality of a capital is strongly decreasing in its distance to the capital. Panel (a) of Figure 7 depicts the bin-scatter relationship between probability of being a part of a nation and the distance to its capital after residualizing for the full set of fixed effects. We see that the the decline in probability of nationality is greatest for small distances; for large distances, in contrast, the probability remains relatively constant (and close to zero). This is consistent with the model prediction that it is changed in the distance to the nearest capitals that are driving changes in nationality, with changes to far-away capitals having no impact on nationality. Hence, these results provide panel evidence consistent with the first equilibrium condition of the framework.

### 3.4.2   Is the capital close to the centroid?

The second equilibrium condition of the framework—equation (5)—makes the prediction that the capital of a nation should be located at the location within a nation that minimizes internal transportation costs. To assess if this true empirically, I will take as given the observed territory of a nation and ask if the location of capitals are indeed close to the model's calculated "centroid" $x_{it}^{centroid}$, i.e. the location which minimizes internal transportation costs:

$$x_{it}^{centroid} \equiv \arg \min_{x \in S_{it}} \int_{S_{it}} d\left(s, x\right) \rho\left(d\right) ds.$$

To do so, I will run regress the distance of a location to its nation's capital on the distance of that location to its nation's centroid and the appropriate set of fixed effects:

$$d\left(s, x_{it}\right)_{st} = \beta d\left(s, x_{it}^{centroid}\right)_{st} + \delta_{it} + \delta_{si} + \varepsilon_{st}, \tag{12}$$

where the nation-year fixed effect $\delta_{it}$ controls for any differences in national productivities and the location-nation fixed effect again ensures that identification arises only from changes in the location of the centroid that arise either due to changes in the territory of the nation or changes in the distribution of resources within the nation. The model predicts the estimation of equation (12) should yield a positive $\beta$: locations that are further away from the national capital should also be further away from the nation's centroid.

Panel (b) of Table 1 presents the results. Consistent with the second equilibrium condition of the framework—and regardless of the measure of distance or the set of fixed effects included—locations that are further away from the centroid of a nation are also further away from the national capital. Panel (b) of Figure 7 depicts the bin-scatter relationship between the distance to the capital and the distance to the centroid; unlike in panel (a), we see the

positive relationship remains consistent throughout all the distribution up until the (few) locations that are very far from the centroid. Hence, as with the first equilibrium condition, these results provide panel evidence consistent with the second equilibrium condition of the framework.

## 3.5 Testing the predictive power of the model

The evidence presented in the previous section demonstrates that the equilibrium conditions of the model are correlated with the observed evolution of national boundaries and capital locations. I ask now whether or not the model is able to predict the future evolution of nations.

I proceed as follows. In each period, I take as given the geography. With this geography, I simulate the model forward one period, taking as its initial conditions the location of capitals a century earlier. I then ask whether the model's predicted change in national boundaries are able to predict the change in borders that actually occurred over that one hundred year period. To do so, I run the following regression, based on equation (11):

$$\mathbf{1}\left\{s \in S_{it}\right\}_{sit} = \beta \mathbf{1}\left\{s \in S_{it}^{model}\right\}_{sit} + \delta_{st} + \delta_{it} + \delta_{si} + \varepsilon_{sit}, \tag{13}$$

where the dependent variable is a dummy variable equal to one if location $s$ is part of nation $i$ in period $t$, the independent variable is a dummy variable equal to one if the model predicts that location $s$ is part of nation $i$ in period $t$, and the fixed effects are as in equation (11); notably, the location-nation fixed effect $\delta_{si}$ ensures that the identification of $\beta$ arises only from locations whose nationality changes.

I simulate two versions of the model. In the first, (and as in the baseline model) I assume all nations are equally productive. In the second, I follow the extension discussed in Section 2.6.1 and allow the productivity of a nation in transporting goods to depend on how large it was the previous century, as in equation (9). Such scale economies could reflect the fact that larger nations are able to better invest in internal infrastructure, reducing future transportation costs.

To estimate the elasticity of productivity to historical nation size, I pursue a two part estimation strategy. First, as discussed in Stylized Fact #1, according to equilibrium condition (4), any point along a border must be equidistant to its two neighboring capitals. If those distance functions differ depending on the productivity of each nation, that productivity can be recovered from the fixed effects of a regression of the (log) distance to one neighboring capital on the (log) distance to the other neighboring capital across all border location points

$b$:

$$\ln d\left(b, x_{it}\right) = \beta \ln \left(b, x_{jt}\right) + \ln \varphi_{it} - \ln \varphi_{jt} + \varepsilon_{bt}.$$

Panel (a) of Online Appendix Figure B.2 (replicating panel (a) of Figure 6) presents the results of this regression. Second, I take these estimated productivities and regress them on the observed size of each nation in the previous period:

$$\ln \hat{\varphi}_{it} = \varepsilon \ln \left\| S_{it-1} \right\| + \nu_{it}.$$

Panel (b) of Online Appendix Figure B.2 presents the results. I find a strong positive relationship between a nation's size one hundred years ago and its productivity today, with an estimated scale economy $\hat{\varepsilon} = 0.12$ (with a standard error of 0.025).

Panel (a) of Table 2 presents the results of the estimation equation (13). The first three columns evaluate the predictive power of the baseline model without scale economies. As is evident, the model is able to successfully predict future changes in the nationality of locations, with positive and statistically significant coefficients regardless of the fixed effects included. Columns (4)-(6) show that the inclusion of scale economies modestly increases the predictive power of the framework. In the most stringent specification in column (6), the regression coefficient implies that the model predicting that a location will changes its nationality to a particular nation makes it 10% more likely that that location will indeed join that particular nation 100 years in the future. Note that this predictive power is despite the fact that the model (by construction) does not predict any entry or exit of nations. Online Appendix Figure B.3 depicts the estimated coefficients over time; as is evident, the predictive power of the model remains consistently strong through 1900 (although, interestingly, the model does not offer statistically significant predictions of the evolution of national boundaries in the 20th century). Hence, it appears that an important force in the evolution of national boundaries is the gradual move toward greater efficiency.

Recall that in the autocratic regime, conflict is predicted to occur only along borders. If the model predicts that a border ought to change its location over the next 100 years, we might then expect conflict would be more likely to occur in the area whose nationality is predicted to change. To assess if this is the case, I calculate the sum of all conflicts in the Historical Conflict Event Dataset (Miller and Bakar, 2023) that occur in each grid cell over each 100 year period. I then regress the number of conflicts that occur in a location in a century on whether or not the model predicted that that location will change its nationality during that century, i.e.:

$$conflict_{st} = \beta \mathbf{1} \left\{ s \in S_{it} \cap s \notin S_{i,t+1}^{model} \right\} + \delta_{it} + \delta_s + \varepsilon_{st}, \tag{14}$$

26

where $\mathbf{1}\left\{ s \in S_{it} \cap s \notin S_{i,t+1}^{model} \right\}$ is a dummy variable equal to one if a location is predicted by the model to change its national identity within the next century, the nation-year fixed effect controls for the overall level of conflict of a given nation (e.g. whether or not the nation is at war); and the location fixed effect ensures identification arises only from changes within location in the degree of conflict (controlling e.g. for whether or not a particular location is militarily strategic).

Before presenting the results, there are two limitations of estimating regression (14). First, conflicts in reality are exceedingly rare: across approximately 2.2 million grid-cell×century pairs, there are only 3,387 observed conficts, i.e. the average number of conflicts in a location in a given century is 0.0015. While this is consistent with the model's predictions that conflict is a measure zero event, it means that it is empirically difficult to predict the locations of conflicts. Second, the prediction that conflicts are more likely to occur along national borders is certainly not unique to the framework presented here. With these caveats in mind, Panel (b) of Table 2 presents the results. As is evident, locations predicted to change their nationality are indeed more likely to experience conflict. Interestingly, this result is no longer statistically significant when including location-nation fixed effects., which is to be expected: if the nationality of a location does not change, the model predicts that conflict should not occur.

## 4   The future evolution of European borders

Having provided some empirical evidence that the theoretical framework can do a good job capturing how geography shapes the evolution of the topography of nations over the past 1000 years, I turn to estimated the future evolution of European borders. The framework developed above can be applied to understand how many different changes in the underlying geography affects the equilibrium shape of nations; examples include changes in the distribution of resources due to agricultural innovations (e.g. the introduction of the potato as in Nunn and Qian (2011)) or changes in the distribution of population (e.g. the Black Death as in Jedwab, Johnson, and Koyama (2022)) or changes in the transportation costs resulting from technological innovations (e.g. the chronometer as in Miotto and Pascali (2022), the steamship as in Pascali (2017), or the rise in air travel as in Feyrer (2019)).

Here, I apply the framework above to understand how changes in agricultural productivity arising from climate change will affect national boundaries. While there has been much work predicting the long-run economic impacts of climate change through changes in patterns of trade, migration, or infrastructure,[14] I am unaware of another attempt to estimate how

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[14]See for example Costinot, Donaldson, and Smith (2016), Balboni (2019), Desmet, Kopp, Kulp, Nagy,

climate change will affect the shape of nations.

To do so, I proceed as in Section 3.5, taking as given the location of capitals in the year 2000 and the geography of the year 2100 and using the framework to predict the equilibrium partition of Europe for the year 2100.[15] To construct the year 2100 geography, I hold constant the transportation costs but update the distribution of resources using the predicted changes in attainable yields for each crop using the GFDL-ESM2M climate change model based on an assumed RCP6.0 (the "higher medium" stabilization pathway). Panel (a) of Figure 8 depicts the estimated change in the distribution of resources relative to the year 2000; as is evident, climate change is predicted to improve agricultural yields throughout much of Europe.

The resulting change in national borders is depicted in panel (b) of Figure 8. The dark blue refers to locations whose nationality is not predicted to change relative to the year 2000. The light blue refers to locations whose nationality is predicted to change even if geography remained constant, i.e. they are locations for whom changing nationality would improve aggregate efficiency absent any change of geography. The yellow locations are those locations which change nationality only because of the changing distribution of resources due to climate change. It is interesting to note that the largest such region where the model predicts a change in nationality is Crimean peninsula and southern Ukraine. The recent conflict in this location is consistent with the evidence provided above that model is able to predict future conflict.

# 5    Conclusion

This paper offers a quantitative framework to understand the equilibrium shape of nations in a world with a rich geography. The framework is based on the principal that national borders evolve to minimize intra-national transportation costs. Such an allocation turns out to be an equilibrium for both autocratic and democratic political economy microfoundations. Despite the richness of the geographies one can consider, under modest conditions, the equilibrium is well behaved and evolves to an efficient steady state. The tractability of the framework also permit a number of extensions, including the endogenous evolution of heterogeneous national productivities and the endogenous entry and exit of nations.

Despite the simplicity of the framework, it matches the observed evolution of Europe over the past millennium surprisingly well. Three stylized facts are broadly consistent with its

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Oppenheimer, Rossi-Hansberg, and Strauss (2021), and Cruz and Rossi-Hansberg (2021).

[15]To allow the updated borders to be affected by the year 2100 resources, I allow the model to predict two periods forward. In the first period, borders are re-drawn given the year 2000 capitals and new capitals are placed at the resulting centroids; in the second period, new borders are drawn based on the location of these centroids.

mechanisms, and the framework performs well in direct tests of the equilibrium conditions using panel variation. Moreover, the framework successfully predicts the future evolution of national boundaries and the locations of future conflicts. When applied to future changes in the geography due to climate change, the framework predicts changing national boundaries in and around the current conflict in Ukraine.

This counterfactual illustrates of how the framework can be brought to bear on many interesting questions about how geography affects the topography of nations. From technological innovations in the cost of shipping to increasing world-wide demand for rare earth metals, changes in geography has the potential to lead to changes in national borders and conflicts between nations. A quantitative framework such as the one presented above offers a useful means of predicting where such changes may arise.

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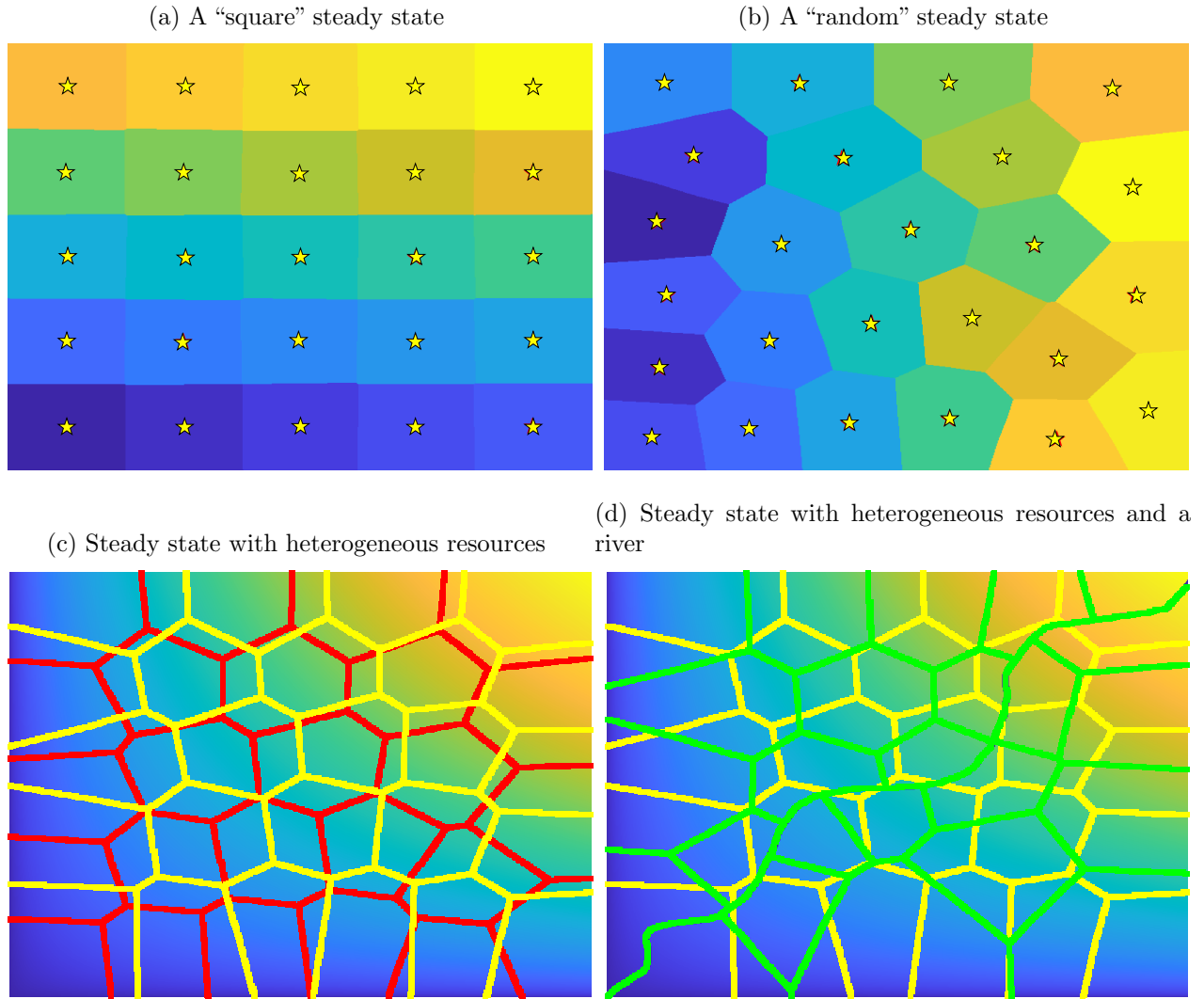TILLY, C. (1992): *Coercion, capital, and European states, AD 990-1992*. Wiley-Blackwell.

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Figure 1: Example: Two countries on a line segment

(a) $t = 1$



(b) $t = 2$



(c) Steady state ($t = 8$)


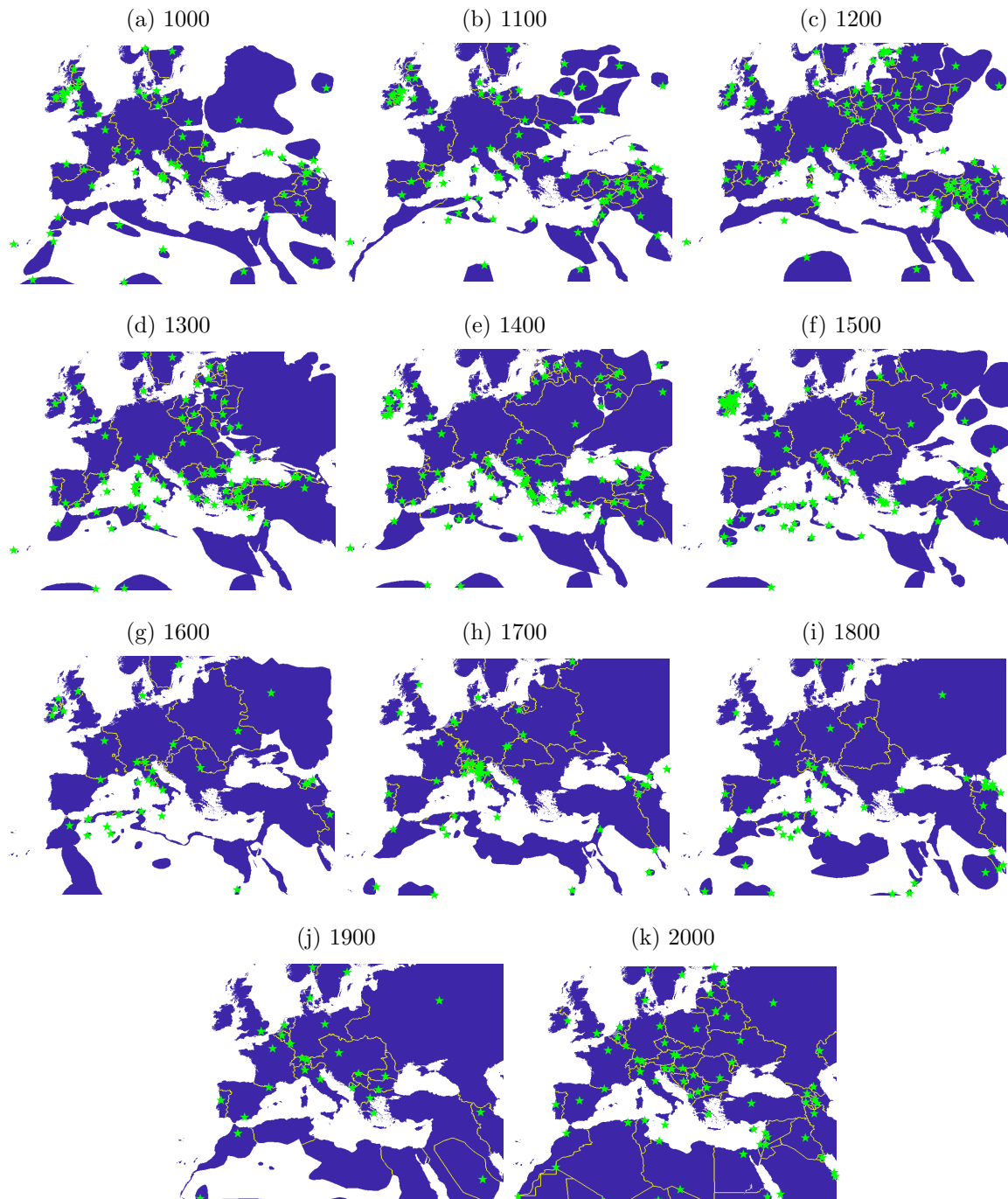
(d) Steady state with a "mountain"
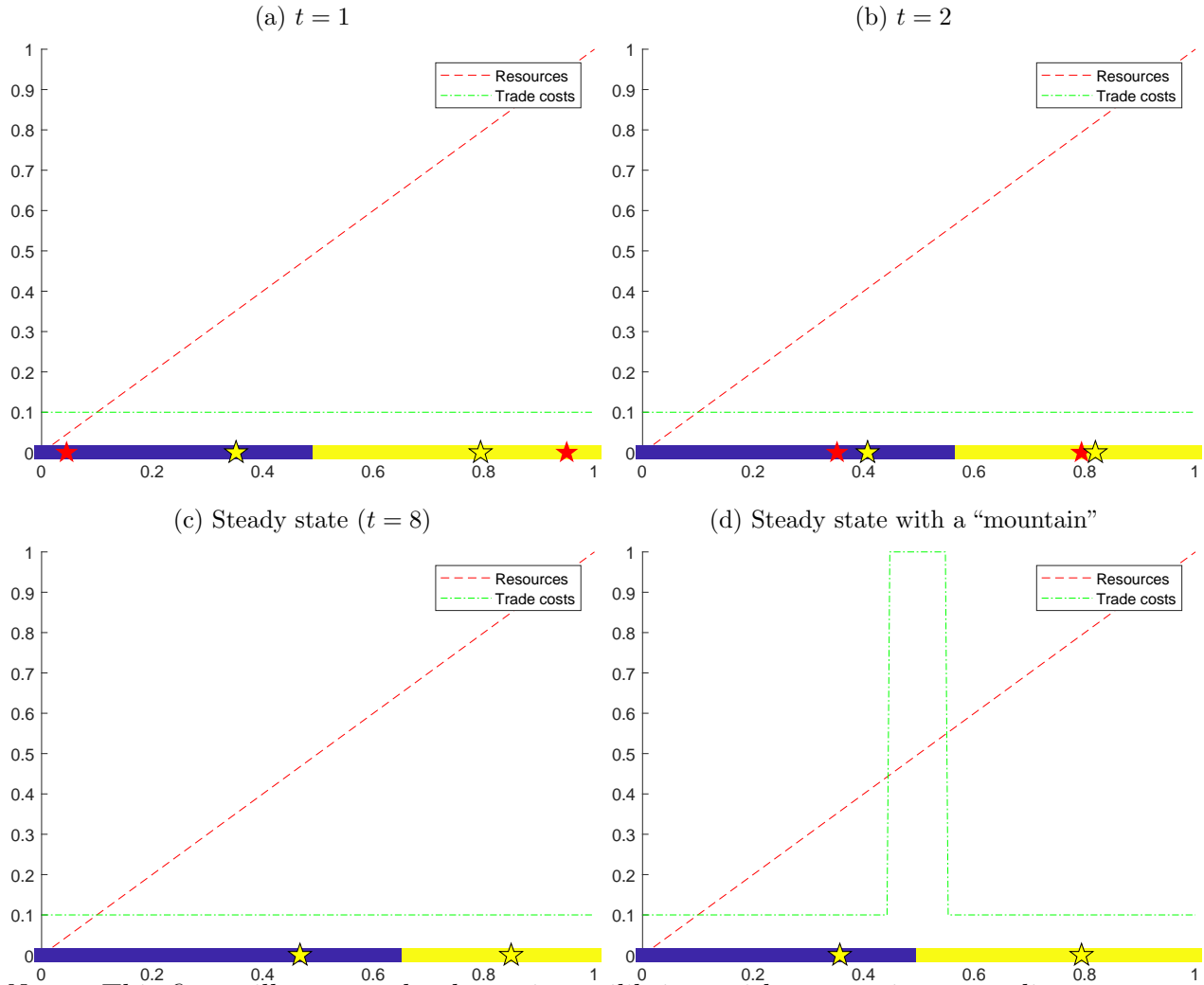


*Notes*: This figure illustrates the dynamic equilibrium with two nations on a line segment with linearly increasing resources. In panels (a)-(c) transportation costs are equal to distance; in panel (d), there is a "mountain" in the middle of the segment which requires additional costs to traverse. Red stars indicate the capital city in the previous period, yellow stars indicate the capital city in the current period, and blue (yellow) indicates the territory of country 1 (2).

Figure 2: Example: Twenty-five countries on a plane

(a) A "square" steady state

(b) A "random" steady state



(c) Steady state with heterogeneous resources

(d) Steady state with heterogeneous resources and a river



*Notes*: This figure illustrates the different possible steady states on a two dimensional square region with 25 countries. Panels (a) and (b) depict steady states with homogeneous resources and initial capital cities arrayed in a grid and randomly, respectively. Panel (c) depicts how the steady state from the random initial capital cities changes if the distribution of resources was heteorgeneous, where the red (yellow) borders indicate the homogenous (heterogeneous) steady state and the blue (yellow) regions indicate low (high) resources. Panel (d) depicts how the steady state would change if there were also a river flowing from the south-west to north-east, with the green indicating the new borders.

Figure 3: European borders and capitals, 1000-2000

(a) 1000        (b) 1100        (c) 1200

(d) 1300        (e) 1400        (f) 1500

(g) 1600        (h) 1700        (i) 1800

(j) 1900        (k) 2000

*Notes*: This figure illustrates the national boundaries and largest city in each nation in Europe from the year 1000 to the year 2000.

Figure 4: Distribution of resources across Europe, 1000-2000

(a) 1000

(b) 1500 (relative to 1000)

(c) 1700 (relative to 1000)

(d) 2000 (relative to 1000)
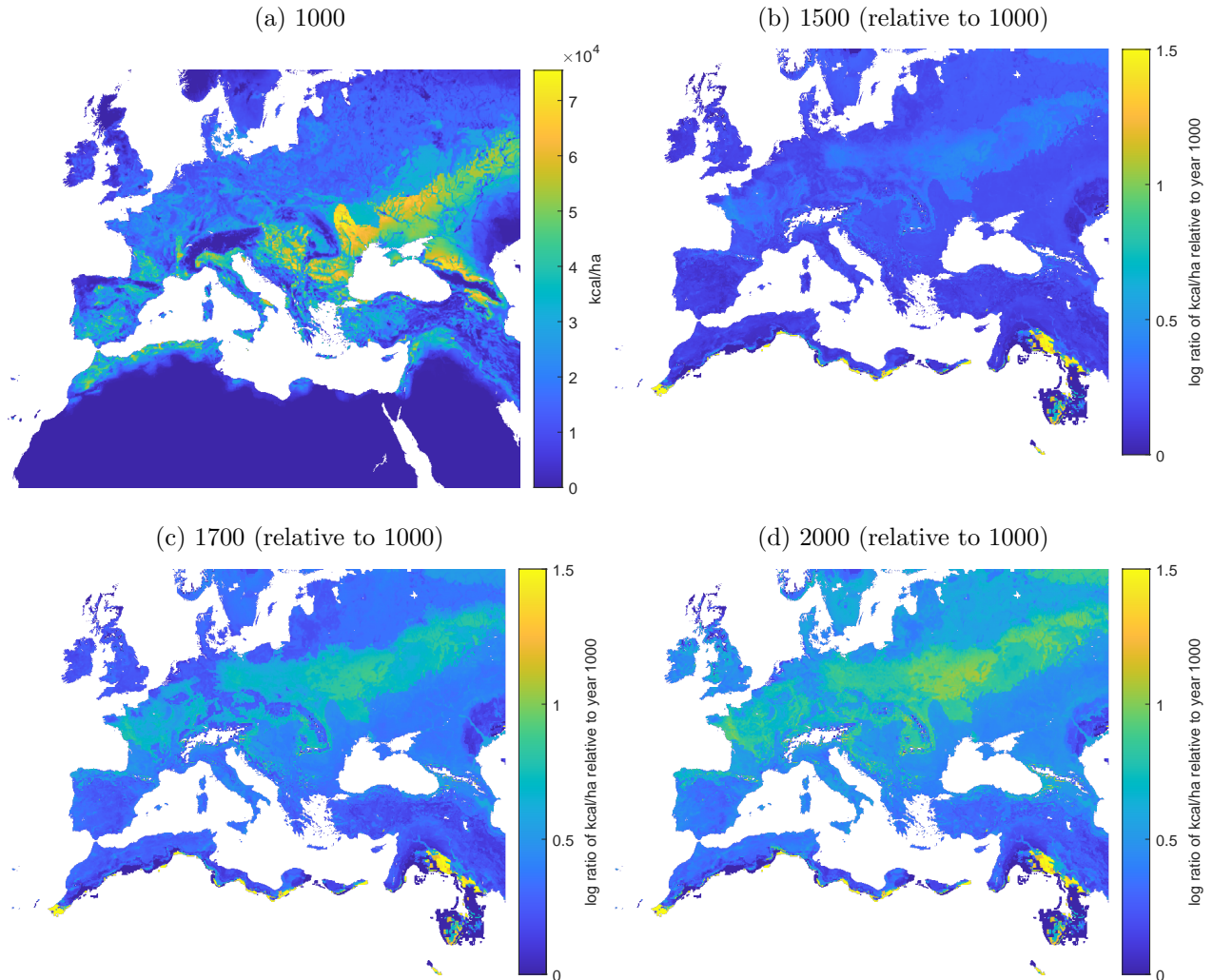


*Notes*: This figure illustrates the spatial distribution of resources, as measured by potential caloric output per hectare given local agroclimatic suitability and the crops that were available for production at the time.

Figure 5: Transportation costs across Europe, 1000-2000

(a) 1000

(b) 1500
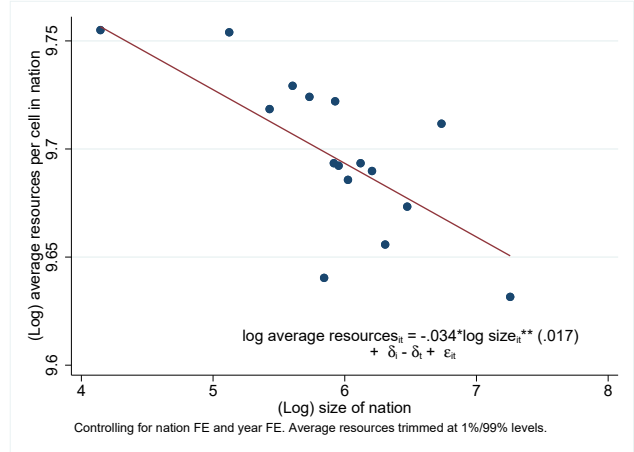
(c) 1700

(d) 2000
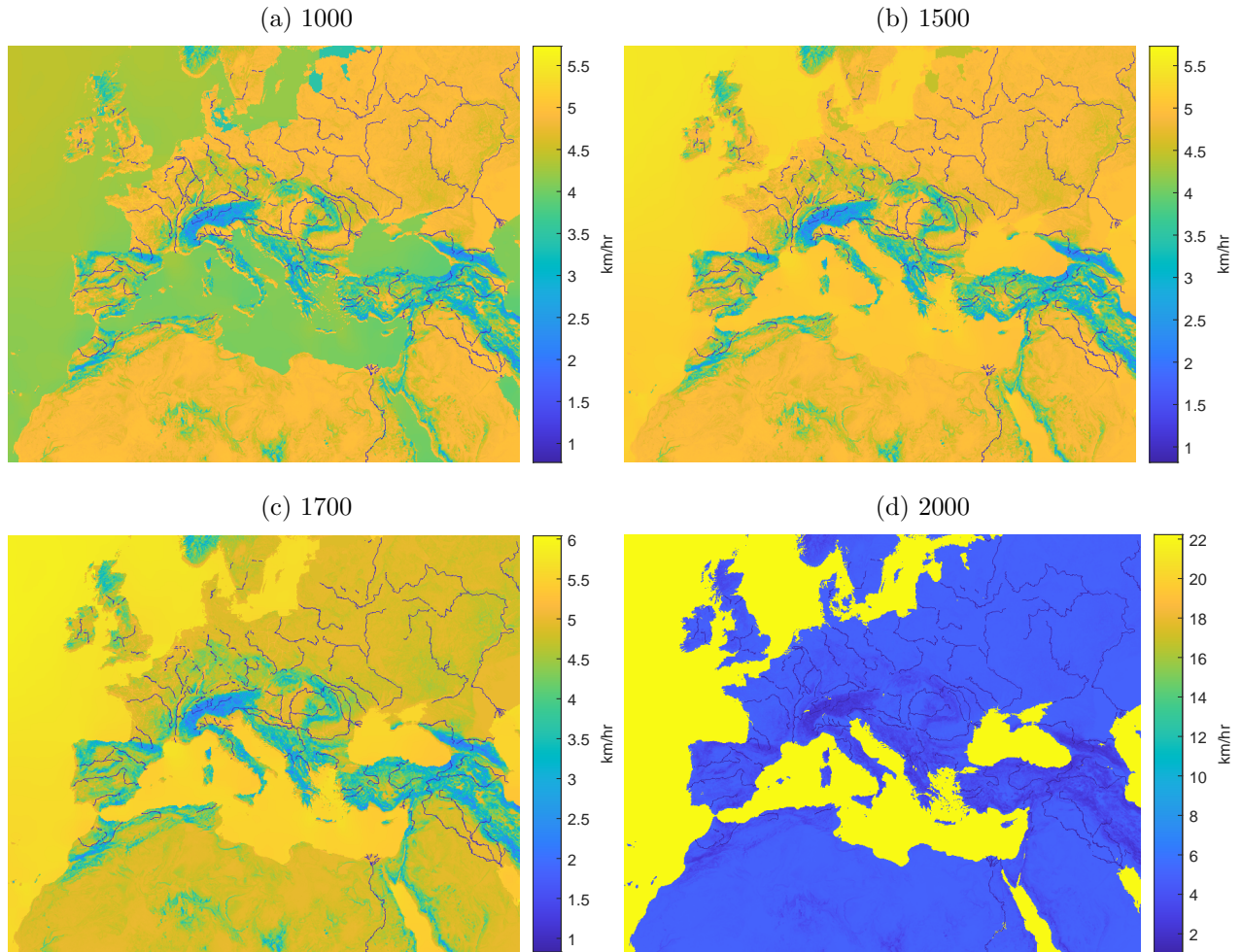
*Notes*: This figure illustrates the speed of travel across Europe, as estimated by the local topography (the prevailing wind speed in the ocean and the ruggedness and presence of Roman roads, rivers, and bridges on land) and the technology available at the time.
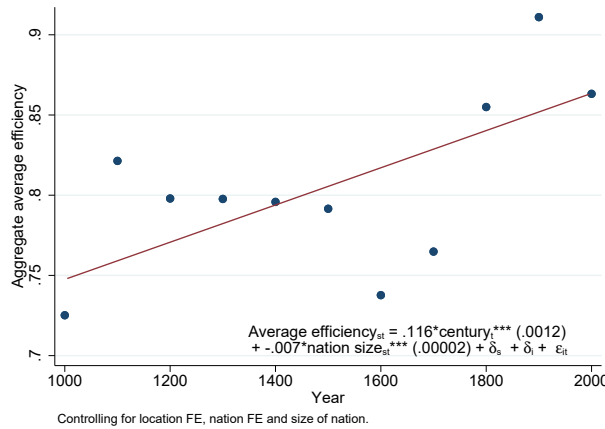
Figure 6: Stylized Facts

(a) Border locations further away from one neighboring capital are also further away from the other neighboring capital
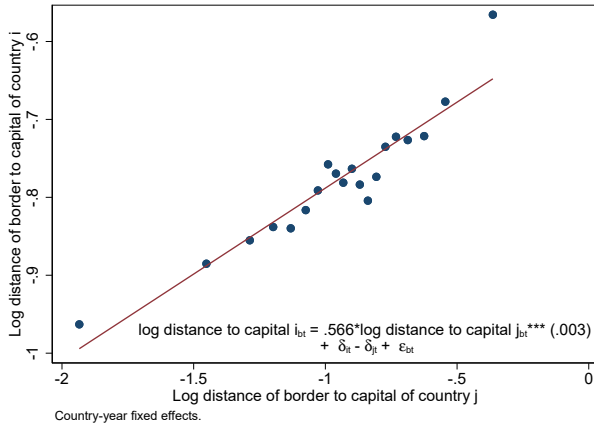
(b) Larger nations lower resources per location



log distance to capital $i_{bt}$ = .566*log distance to capital $j_{bt}$*** (.003) + $\delta_{it}$ - $\delta_{jt}$ + $\varepsilon_{bt}$

Country-year fixed effects.

log average resources$_{it}$ = -.034*log size$_{it}$** (.017) + $\delta_i$ - $\delta_t$ + $\varepsilon_{it}$

Controlling for nation FE and year FE. Average resources trimmed at 1%/99% levels.

(c) National boundaries have become more efficient over time



Average efficiency$_{st}$ = .116*century$_t$*** (.0012) + -.007*nation size$_{st}$*** (.00002) + $\delta_s$ + $\delta_i$ + $\varepsilon_{it}$

Controlling for location FE, nation FE and size of nation.

*Notes*: This figure presents three stylized facts. Panel (a) shows that on average a border location that is closer to one neighboring nation's capital is also closer to the other neighboring nation's capital. Panel (b) shows that on average larger nations comprise locations with lower resources. Panel (c) shows that on average the transportation costs incurred by a location have declined over time. Each panel depicts both the bin-scatter and regression line, along with the regression coefficients. In panel (a), a unit of observation is a location on a border and the regression specification controls for nation-year fixed effects. In panel (b), a unit of observation is a nation in a given year and the sample excludes nations which are not fully contained within greater Europe and those below 1% or above 99% average resources. The regression includes nation and year fixed effects. In panel (c), a unit of observation is a location in a given year and the regression includes location and nation fixed effects, as well as a control for nation size.

Figure 7: Testing the equilibrium conditions

(a) Is nationality determined by the nearest capital?
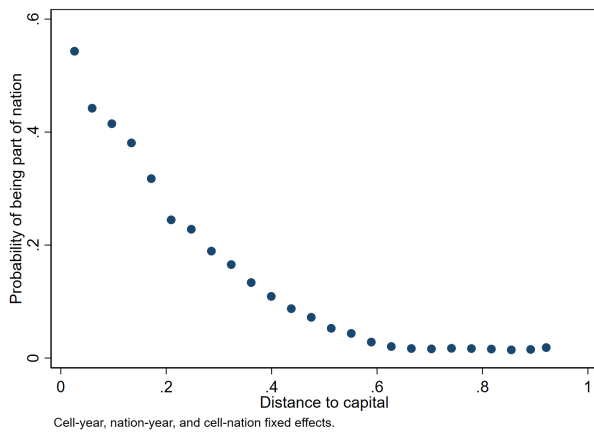
(b) Is the capital close to the centroid?
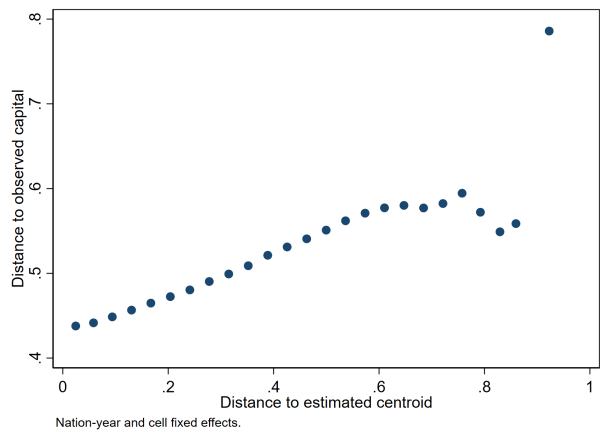


*Notes*: This figure depicts bin-scatters of the regressions testing the two equilibrium conditions of the framework. Panel (a) shows that a location is more likely to be a part of a nation the closer it is to the observed capital, conditional on cell-year, nation-year, and cell-nation fixed effects. Panel (b) shows that locations within a nation that are close the observed capital are also closer to the estimated centroid of the nation, conditional on nation-year and cell fixed effects.

Figure 8: How will climate change affect the future evolution of national boundaries?

(a) Estimated change in resources in 2100 (relative to 2000)
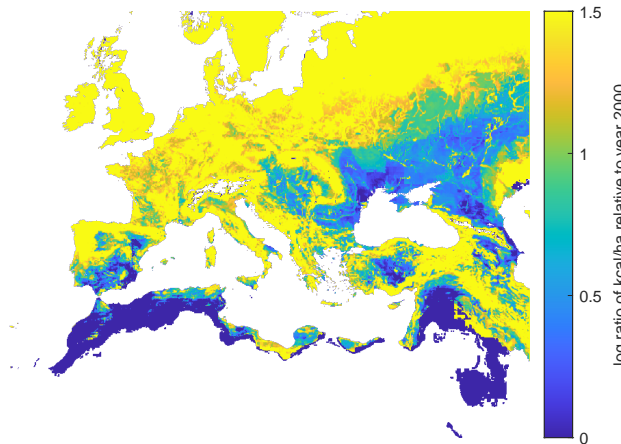
(b) Estimated change in borders in 2100
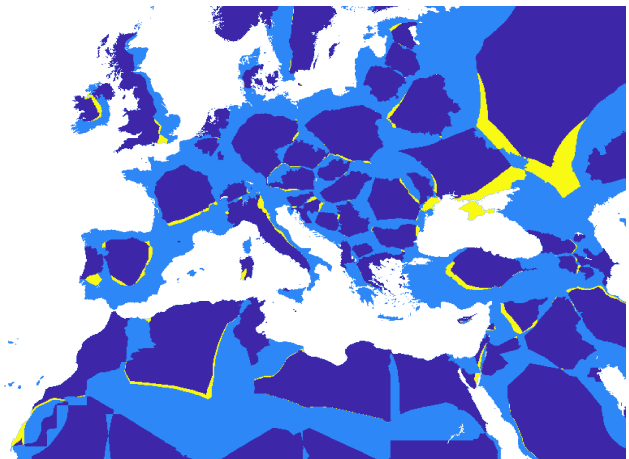


*Notes*: This figure depicts the estimated impact of climate change on European borders. Panel (a) depicts the change in the spatial distribution of resources between 2000 and 2100, as estimated in the change in potential caloric output per hectare given local agroclimatic suitability using the GFDL-ESM2M climate model based on RCP6.0. Panel (b) depicts the resulting estimated change in national boundaries. Dark blue indicates regions where boundaries are unchanged, light blue indicates regions where the framework estimates boundaries would change even in the absence of climate change, and yellow indicates regions where the framework estimates boundaries would change only in the presence of climate change.

Table 1: TESTING THE EQUILIBRIUM CONDITIONS

Panel (a): Is nationality determined by the nearest capital?

| | Fraction value lost | | | Log travel time | | |
| --- | --- | --- | --- | --- | --- | --- |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Distance to the capital | -0.463*** | -0.481*** | -0.256*** | -0.071*** | -0.076*** | -0.041*** |
| | (0.002) | (0.002) | (0.007) | (0.000) | (0.000) | (0.001) |
| Cell-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Nation-year FE | No | Yes | Yes | No | Yes | Yes |
| Cell-nation FE | No | No | Yes | No | No | Yes |
| R-squared (within) | 0.129 | 0.141 | 0.008 | 0.102 | 0.115 | 0.006 |
| Clusters | 12756 | 12756 | 11864 | 12756 | 12756 | 11864 |
| Obs. (millions) | 6.6 | 6.6 | 4.9 | 6.6 | 6.6 | 4.9 |

Panel (b): Is the capital close to the centroid?

| | Fraction value lost | | | Log travel time | | |
| --- | --- | --- | --- | --- | --- | --- |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Distance to centroid | 0.285*** | 0.229*** | 0.070*** | 0.275*** | 0.210*** | 0.056*** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| Cell FE | No | Yes | Yes | No | Yes | Yes |
| Nation-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Cell-nation FE | No | No | Yes | No | No | Yes |
| R-squared (within) | 0.071 | 0.041 | 0.004 | 0.066 | 0.037 | 0.003 |
| Clusters | 318621 | 296596 | 265504 | 318617 | 296585 | 265482 |
| Obs. (millions) | 2.5 | 2.5 | 2.0 | 2.5 | 2.5 | 2.0 |

*Notes:* Ordinary least squares. In panel (a), each observation is a 10km×10km grid cell-nation pair in a particular century, and the dependent variable is a dummy variable equal to one if the grid cell is part of the nation in that century. In panel (b), each observation is a 10km×10km grid cell in a particular century and the dependent variable is the distance to the capital city. In panel (a), there is 4% sample of grid cells; in panel (b), all grid cells are included. In both panels, standard errors clustered at the grid cell reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

Table 2: Can the model predict the evolution of nations?

Panel (a): Can the model predict future nationality?

|  | No scale economies | | | Scale economies | | |
| --- | --- | --- | --- | --- | --- | --- |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
| Predicted nationality | 0.556*** | 0.530*** | 0.081*** | 0.625*** | 0.600*** | 0.099*** |
|  | (0.003) | (0.003) | (0.002) | (0.003) | (0.003) | (0.002) |
| Cell-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Nation-year FE | No | Yes | Yes | No | Yes | Yes |
| Cell-nation FE | No | No | Yes | No | No | Yes |
| R-squared (within) | 0.211 | 0.201 | 0.008 | 0.284 | 0.268 | 0.011 |
| Clusters | 12756 | 12756 | 11713 | 12756 | 12756 | 11713 |
| Obs. (millions) | 6.1 | 6.1 | 4.5 | 6.1 | 6.1 | 4.5 |

Panel (b): Can the model predict future conflict?

|  | No scale economies | | | Scale economies | | |
| --- | --- | --- | --- | --- | --- | --- |
|  | (1) | (2) | (3) | (4) | (5) | (6) |
| Predicted to change hands | 0.478*** | 0.262*** | 0.123 | 0.570*** | 0.220** | 0.121 |
|  | (0.080) | (0.091) | (0.128) | (0.088) | (0.109) | (0.162) |
| Nation-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Cell FE | No | Yes | Yes | No | Yes | Yes |
| Cell-nation FE | No | No | Yes | No | No | Yes |
| R-squared (within) | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Clusters | 296599 | 269209 | 251309 | 296599 | 269209 | 251309 |
| Obs. (millions) | 2.2 | 2.2 | 1.7 | 2.2 | 2.2 | 1.7 |

*Notes:* Ordinary least squares. In panel (a), each observation is a 10km×10km grid cell-nation pair in a particular century, and the dependent variable is a dummy variable equal to one if the grid cell is part of the nation in that century. In panel (b), each observation is a 10km×10km grid cell in a particular century, the dependent variable is the number of conflicts in a grid cell over the next 100 years, and the independent variable is a dummy variable equal to one if the model predicts in the next period a grid cell will change its national affiliation and is rescaled by 1,000 for readability. In panel (a), there is 4% sample of grid cells; in panel (b), all grid cells are included. In both panels, standard errors clustered at the grid cell reported in parentheses. Stars indicate statistical significance: * p<.10 ** p<.05 *** p<.01.

# The Topography of Nations: Online Appendix

## Treb Allen

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# A Theory Appendix

## A.1 Proofs

### A.1.1 Proof of Proposition 1

We first restate the Proposition:

**Proposition 1.** *For any geography $\{\rho, d\}$ and capital cities $\{x_{i,t-1}\}_{i \in \mathcal{N}}$, there exists unique territories $\{S_{it}\}$ satisfying equation (4).*

*Proof.* Because $d$ is bounded and $S$ is compact, for any $s \in S$ and any $\{x_{i,t-1}\}_{i \in \mathcal{N}}$ the finite set $\{d(s, x_{it-1})\}_{i \in \mathcal{N}}$ has a minimum, so existence is assured. Moreover, because $d$ is a distance metric, any points $s \in S$ on the boundary of two sets (i.e. $\exists i, j \in \{1, ..., N\}$ such that $d(s, x_{it-1}) = d(s, x_{jt-1}) = \min_{k \in \mathcal{N}} d(s, x_{kt-1})$) are measure zero, as any perturbation from $s$ in a non-orthogonal direction to the least cost path between $x_{it-1}$ and $x_{jt-1}$ will not be on the boundary, so that $S_{it} \cap S_{jt} = \emptyset$, i.e. uniqueness is assured. $\qquad\square$

### A.1.2 Proof of Proposition 2

We first restate the Proposition:

**Proposition 2.** *For any geography $\{\rho, d\}$, there exists a $\{x_{it}\}_{i \in \mathcal{N}}$ satisfying equation (5). If Condition 1 holds and $\{S_{it-1}\}_{i \in \mathcal{N}}$ are convex, the $\{x_{it}\}_{i \in \mathcal{N}}$ satisfying equation (5) are unique.*

*Proof.* This proof follows Remark 3.6 of Du, Faber, and Gunzburger (1999). Since the function $\int_{S_{it-1}} \rho(s) d(s, x) ds$ is continuous and $S_{it-1}$ is compact, it obtains its minimum, so existence is assured. Uniqueness is proved by contradiction. Suppose there exist two solutions $x_{it}$ and $y_{it}$ that each satisfy (5), i.e. $\int_{S_{it-1}} \rho(s) d(s, x_{it}) ds = \int_{S_{it-1}} \rho(s) d(s, y_{it}) ds \leq \int_{S_{it-1}} \rho(s) d(s, v) ds$ for all $v \in S$. Define $z \equiv \lambda x_{it} + (1 - \lambda) y_{it}$, where the $\lambda$ is chosen from Condition 1. Because $S_{it-1}$ is convex, $z \in S_{it-1}$. Then from Condition 1 we have $\int_{S_{it-1}} \rho(s) d(s, z) ds < \int_{S_{it-1}} \rho(s) d(s, x_{it}) ds$, a contradiction. $\qquad\square$

### A.1.3 Proof of Proposition 3

We first restate the Proposition:

**Proposition 3.** *For any geography and $\{x_{i0}\}_{i \in \mathcal{N}}$, aggregate welfare improves over time, i.e. $R_t \geq R_{t-1} \; \forall t$.*

*Proof.* Consider first the case when $t$ is even. From equation (5), we have for all $x \in S_{it-1}$

$$\int_{S_{it-1}} \rho(s) d(x_{it}, s) ds \leq \int_{S_{it-1}} \rho(s) d(x, s) ds$$

so that:

$$\int_{S_{it-1}} \rho(s) d(x_{it}, s) ds \leq \int_{S_{it-1}} \rho(s) d(x_{it-1}, s) ds$$

and so, by summing over all $i \in \mathcal{N}$ we immediately find $R_t \geq R_{t-1}$.

Now consider the case when $t$ is odd. We proceed by contradiction. Suppose that $R_t < R_{t-1}$. From the definition of efficiency, this implies that there exists an $s \in S$ such that $s \in S_{it}$ and $s \in S_{jt-1}$ where $i \neq j$ such that $\rho(s) d(x_{jt-1}, s) < \rho(s) d(x_{it-1}, s)$. But by equation (4), $d(x_{it-1}, s) \leq d(x_{jt-1}, s)$, which is a contradiction. $\square$

### A.1.4 Proof of Proposition 5

We first restate the Proposition:

**Proposition 5.** *Any steady state is locally efficient, i.e. deviations from either $S_i^*$ or $x_i^*$ holding the other constant cannot improve the aggregate welfare.*

*Proof.* The argument closely follows the proof of Proposition (3). Let $\{x_i^*, S_i^*\}_{i \in \mathcal{N}}$ be a steady state and define $R(\{x_i\}, \{S_i\}) \equiv \sum_{i \in \mathcal{N}} \int_{S_i} \rho(s)(1 - d(x_i, s)) ds$. Consider first a deviation from $\{x_i^*\}$ to $\{\tilde{x}_i\}$, holding $\{S_i^*\}$ constant. From equation (7), we immediately have:

$$\int_{S_i^*} \rho(s) d(s, x_i^*) ds \leq \int_{S_i^*} \rho(s) d(s, \tilde{x}_i) ds,$$

so that $R(\{x_i^*\}, \{S_i^*\}) \geq R(\{\tilde{x}_i\}, \{S_i^*\})$. Consider now a deviation from $\{S_i^*\}$ to $\left\{\tilde{S}_i\right\}$, holding $\{x_i^*\}$ constant. Suppose that $R\left(\{x_i^*\}, \left\{\tilde{S}_i\right\}\right) > R(\{x_i^*\}, \{S_i^*\})$. This means that there exists an $s \in S$ such that $s \in \tilde{S}_i$ but $s \in S_j^*$ for $j \neq i$ such that $\rho(s) d(x_i^*, s) < \rho(s) d(x_j^*, s)$. But from equation (6), if $s \in S_j^*$ we have $d(x_j^*, s) \leq d(x_i^*, s)$, a contradiction. $\square$

## A.2 Additional Extensions

### A.2.1 Economies of scale

In the baseline model, there are no economies of scale in nation size. In reality, residents of larger nations may benefit from having the fixed costs of public good provision divided across a greater population. Consider a democratic regime where a nation finances the cost of its public good by imposing a lump sum tax of $T_i$ on each resident, where we assume $T_i$ is declining in the size of the nation in the prior period. Then the payoff of a resident of location $s \in S$ who chooses to be part of nation $i$ is:

$$R_{it}(s) = 1 - d(s, x_{it}) - T_i.$$

As in the baseline model, an individual residing in $s$ in odd $t$ who is choosing which nation to join, i.e. $\max_{i \in \mathcal{N}} R_{it}(s)$, but this will now lead to the following equilibrium territory of nation $i$:

$$S_{it} = \left\{ s \in S | d(s, x_{it}) + T_i \leq \min_{j \in \mathcal{N}} d(s, x_{jt}) + T_j \right\}. \tag{A.1}$$

In the special case where $d(s, x) \equiv \|s - x\|_2^2$, equation (A.1) generates what is known as an *additively weighted Voronoi diagram*; see e.g. Section 3.1.2 of Boots, Okabe, and Sugihara

([1999](#)). Intuitively, individuals that are equidistant between two possible nations will prefer to inhabit the nation with the lower lump sum tax.Note the similarity between this framework and the extension incorporating heterogeneous nation productivity in Section 2.6.1 with persistent economies of scale as in equation (9): both generate a force whereby historically large nations continue to grow and differ only in the particular functional form.

### A.2.2 Unclaimed territories

In the baseline model, all locations $s \in S$ in all periods are part of some nation. This is because the transportation costs are assumed to take an iceberg form, implying that for any (finite) transportation cost, there is positive value to every nation from every location. Historically, however, not all territory was part of a nation. Extending the framework to include unclaimed territory can be done in a straightforward way by assuming that agents (either feudal lords or farmers) must incur a fixed cost $f$ of territorial acquisition in addition to the (variable) transportation costs. As a result, all locations $s \in S_{0,t}$ would be unclaimed, where:

$$S_{0,t} \equiv \left\{ s \in S | \rho\left(s\right) \left( 1 - \min_{i \in \mathcal{N}} d\left(s, x_{it}\right) \right) < f \right\}.$$

As long as capital cities are allowed to be constructed on unclaimed territories, introducing such unclaimed territories does not change the equilibrium properties of the model, as territory of each could be calculated as if there are no fixed costs and then any empty territory falling within the convex hull of the territory could be treated as if $\rho\left(s\right) = 0$.

### A.2.3 Multiple markets

In the baseline model, each nation chooses a single capital city. In principle, the model could be extended to allow each nation $i$ to choose $K_i$ different markets by modifying the dynamic equilibrium as follows:

$$S_{it} = \left\{ s \in S | \min_{k \in \{1, \dots, K_i\}} d\left(s, x_{i,k,t-1}\right) \leq \min_{j \in \{1, \dots, N\}} \min_{k \in \{1, \dots, K_j\}} d\left(s, x_{j,k,t-1}\right) \right\}$$

$$\{x_{i,k,t}\}_{k \in \{1, \dots, K_i\}} = \arg \min_{\{x_k\} \in S_{it-1}^{K_i}} \int_{S_{it-1}} \rho\left(s\right) \min_{k \in \{1, \dots, K_i\}} d\left(x_k, s\right) ds.$$

While such an extension does not affect the equilibrium properties of the dynamic equilibrium, in practice the computational burden of solving for the equilibrium location of the multiple cities that minimizes the resources lost to transportation costs within a nation increases exponentially with $K_i$.

### A.2.4 Heterogeneous goods

In the baseline model, the payoff to a nation is the sum of resources from all locations within the territory net of transportation costs. Implicitly, different location's resources are perfect substitutes. In reality, however, there may be more subtle patterns of substitution of resources across locations.

To incorporate such patterns of substitution, assume that nations aggregate resources across locations using a constant elasticity of substitution (CES) aggregator:

$$R\left(x_{it}, S_{it}\right) = \left(\int_{S_{it}} \left(\rho\left(s\right)\left(1 - d\left(s, x_{it}\right)\right)\right)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 0$ is the elasticity of substitution. The baseline model is a special case of this extension where $\sigma \to \infty$. In this case, the marginal benefit of each $s \in S_{it}$ to the nation is:

$$\lambda_i \left(\rho\left(s\right)\left(1 - d\left(s, x_{it}\right)\right)\right)^{\frac{\sigma-1}{\sigma}},$$

where $\lambda_i \equiv \frac{\sigma}{\sigma-1}\left(\int_{S_{it}} \left(\rho\left(s\right)\left(1 - d\left(s, x_{it}\right)\right)\right)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{1}{\sigma-1}}$ captures the shadow value to nation $i$ of increasing its territorial holdings. As a result, equations (4) and (5) of the dynamic equilibrium become:

$$S_{it} = \left\{ s \in S \Big| \frac{1 + T\left(s, x_{it-1}\right)}{\lambda_i^{\frac{\sigma}{\sigma-1}}} \leq \min_{j \in \{1,...,N\}} \frac{1 + T\left(s, x_{jt-1}\right)}{\lambda_j^{\frac{\sigma}{\sigma-1}}} \right\} \tag{A.2}$$

$$x_{it} = \arg \min_{x \in S_{it-1}} \left(\int_{S_{it-1}} \left(\rho\left(s\right)\left(1 - d\left(s, x\right)\right)\right)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}. \tag{A.3}$$

Note the the similarity between equation (A.2) defining the territory here and equation (8) defining the territory in the presence of heterogeneous nations. The difference is that here the productivity of a nation is determined endogenously by its shadow value of acquiring territory.
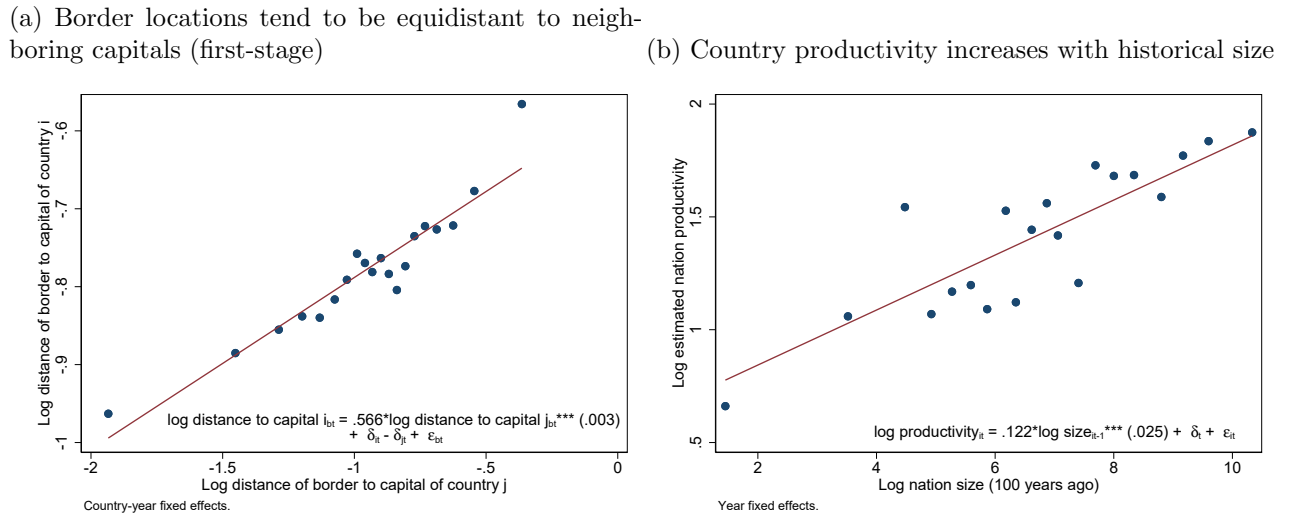
# B  Additional Tables and Figures

Figure B.1: Estimated sailing technology, 1000-2000

(a) Intercept (km/hr with no wind)

(b) Wind coefficient (km/hr per km/hr of wind)
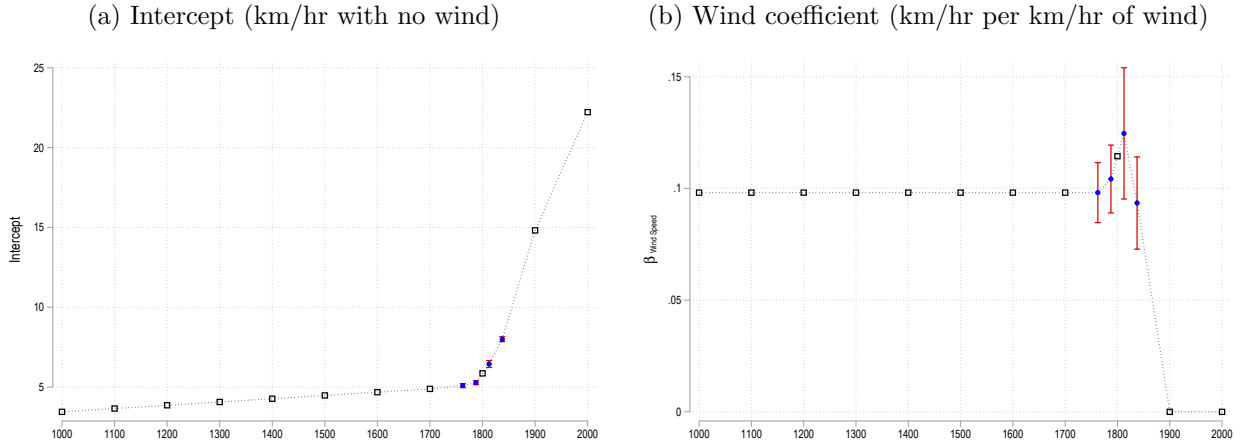


*Notes*: This figure illustrates estimated sailing technology between the years 1000 and 2000. The estimates come from regressions of observed speed of travel on the average prevailing wind speeds along route using log-book data between 1700-1850 and historical estimates of sailing technology from Whitewright (2012).

Figure B.2: Estimating the elasticity of nation productivity to its historical size

(a) Border locations tend to be equidistant to neighboring capitals (first-stage)

(b) Country productivity increases with historical size



log distance to capital $i_{bt}$ = .566*log distance to capital $j_{bt}$*** (.003) + $\delta_{it}$ - $\delta_{jt}$ + $\varepsilon_{bt}$

Country-year fixed effects.

log productivity$_{it}$ = .122*log size$_{it-1}$*** (.025) + $\delta_t$ + $\varepsilon_{it}$

Year fixed effects.

*Notes*: This figure depicts the estimated relationship between nation productivity and its historical size. Panel (a) depicts a bin-scatter showing a strong positive relationship between the distance of a border location to one its neighboring capitals and the distance to its other neighboring capital; the nation-year fixed effects of this regression are estimates of each nation's productivity. Panel (b) depicts a bin-scatter showing a strong positive relationship between these estimated productivities and the size of that nation one hundred years prior. The estimated elasticity of 0.122 is used when assessing the predictive power of the model in columns (4)-(6) of Table 2.

Figure B.3: How does the predictive power of the model change over time?

(a) No scale economies
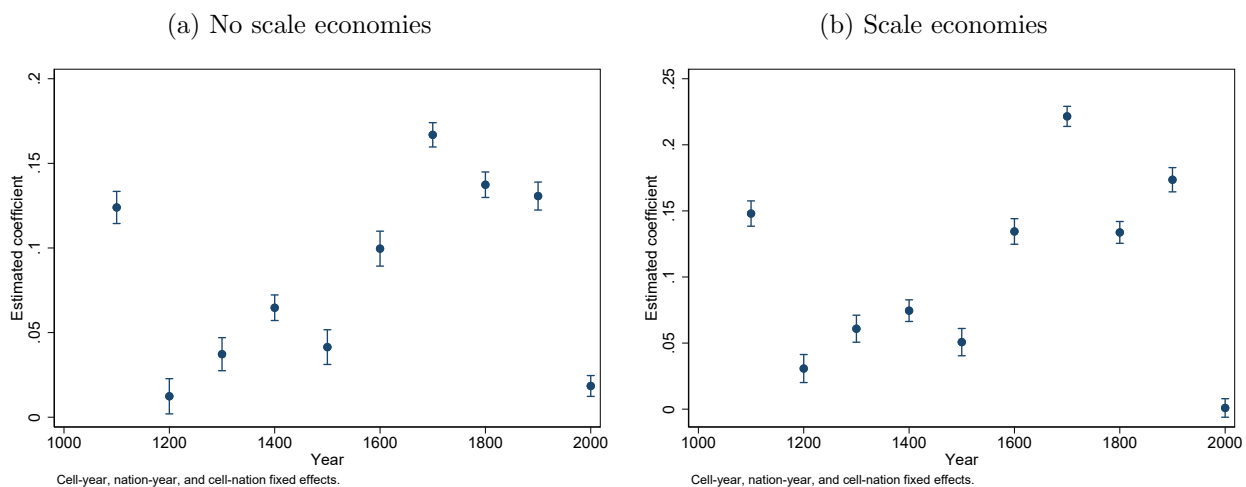
(b) Scale economies



*Notes*: This figure presents the estimated predictive power of the model from regression equation (13) for each century separately. Panel (a) comes from the model with homogeneous national productivities; panel (b) allows for scale economies.